ARITHMETIC

IN TWO PARTS:

Containing

I. *A Syſtem of the Art*, in Memorial Verſes, and Dictionary-wiſe; for the readier Learning, Conſulting, and Recollecting of the ſeveral articles: The whole more Commodious than any, and more Comprehenſive than all, that have yet been calculated for the Uſe of SCHOOLS.

II. *A Collection of Exerciſes*, accommodated to the various Occaſions of Buſineſs, and contrivd for the Eaſe of Teachers: being diſpoſd, partly (under each rule) Alphabetically, for the readier recourſe; partly (under all) Promiſcuouſly, for the ſeverer TRIAL.

By SOLOMON LOWE,

Schoolmaſter, at Hammerſmith. — R

Numbers are ſo much the meaſure of every thing that is valuable; that it is not poſſible to demonſtrate the ſucceſs of any action, or the prudence of any undertaking, without them. ADDISON, Spectat. n. 174.

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PREFACE.

IF the reader ſhall find *the title* of this piece *verified by the work* (which it was deſignd he ſhould) there will be no need of an *Apology* for adding this ſyſtem to the multitudes of arithmetics, that have been already publiſht.——I have, therefore, only to add, in order to give the reader a more preciſe idea of what I have aimd-at, that 1

The Directions, which I have given for the performance of the operations, and the application of the rules; are contrivd to begin with the name of each of them; the better to ſtart the train of ideas, that will naturally ariſe after the ſound of the leading word.——How eaſily, above all things, the doctrin of numbers ſlips out of the mind; every body, that is not in conſtant practiſe of it, muſt be very ſenſible. The beſt ſecurity againſt this uncertainty ſeems to be a ſet of rules, quite detacht from all examples, and obſervations thereupon; which are apt to embarraſs the mind, and leave it at a loſs (for want of due diſtinction) how much the memory ought to be charg'd with. I have therefore (hereby) provided againſt this inconvenience: and not only given the rules diſtinct, and full enough for fixing the idea of what they are deſignd to teach: (V. paragraph 9) but have thrown them into verſe: that they may be the more eaſily learnt, the more firmly retaind, and the more readily recollected, on any occaſion.——Thus advantageous

will

will this peculiarity be to the learner; and, conſequently, to the teacher: beſide that it will ſave him the fruitleſs drudgery of writing fair, in every copybook, ſome imperfect ſketches of tables, and directions under every head of rules and operations; which is now the common method: though, for ought I can ſee, it anſwers no other purpoſe than that of making a ſhow of their dexterity in writing, ſtriking, and flouriſhing; which has nothing to do with the art it ſelf; and can be of no value but to amuſe and dazzle the illiterate. 2

In the Verſes (which have the cadence of latin hexameters) the accent, which is added to aſcertain the right reading of them, denotes the firſt ſyllable of a dactyl; or that the two following ſyllables are to be pronounc'd ſhort. 3

As to the Notes (which are chiefly exemplifications of the text) they are ſo layd-out as to ſtrike the eye, for the readier reference, and eaſier application. 4

For Particulars (beſide ſeveral things *new* in the moſt conſiderable articles) care has been taken, from a great number of the beſt ſyſtems, to *ſelect* the readieſt ways, and clevereſt compendiums. 5

The Compaſs of the whole is in 134 verſes, including 23 for the account of *coins, weights, and meaſures*; which (beſide the advantage of being eaſily got by heart, and certainly rememberd) do completely anſwer all the purpoſes of the largeſt tables; and may more ſafely be depended-on than the beſt. And I will venture to add that they contain a great deal more than is to be met-with in any, or (indeed) all the ſyſtems, that have been hitherto publiſht.——But *the Whole of what is commonly taught in ſchools, and All that the generality of people, in buſineſs, have occaſion for* (to-wit, the four operations, with the rules of Reduction, Proportion, and Practiſe) are fully taught in about 50 verſes; which it will be very convenient, as it is very eaſy, to get-by-heart———For *the Reſt*, which there is much leſs occaſion for in ordinary affairs, the reader (as he ſhall like beſt) may . . either charge his memory with the rules . . or familiarize the practiſe by dint of application, in the ſame manner as in other ſyſtems, according to the common uſage. 6

As to the Plan of the work, it is drawn-up in the mnemonic form; a method not yet generally receivd: but, whatever prejudices may ariſe in people's minds againſt it, on account of its novelty; it will be found, upon trial, both eaſy, and uſeful.——By the key to the art, as exhibited in the frontiſpiece, it is evident that a child may be made maſter of it, in leſs than half an hour's time; and I can aſſure the reader, from certain experience, that, how difficult and forbiding-ſoever the jargon of it may appear; nothing will ſtick more effectually in the memory, when once familiarizd by frequent repetition, ſo as to flow into the mind without reflexion.—Beſides, that, without the leaſt encumbrance to the mind, or prejudice to other acquiſitions, even children (by the mere dint of ſound) may hereby be enabled to lay-up a large treaſure of uſeful notices; ſo as to be furniſht (betimes) with a ſatisfactory certainty, readineſs, and exactneſs, in things, of which maſters themſelves, and men of reading have generally but an imperfect and confus'd remembrance. It is a very eaſy matter, however impracticable it may ſeem, for a boy of but indifferent parts to be perfectly acquainted-with, and readily to tell any body, the diſtances of the moſt conſiderable places on the face of the earth; the diameters, magnitudes, revolutions, and velocities of the heavenly bodies; all the epochas in chronology; the remarkables in hiſtory, &c. in ſhort to be poſſeſt of a fine variety of principles in the ſeveral ſciences; for the greateſt part of which even profeſſors are under a neceſſity, from time to time (on any occaſion, in controverſy, or converſation) to have recourſe to books, or (when thoſe are not at hand) to be diſappointed; how much ſoever it may be to their diſcredit, or prejudice. 7

The Diſpoſition of the parts is dictionary-wiſe, for the readier reference to any particular: an advantage of more conſequence to ſave trouble than (perhaps) is generally imagind. I am the more ſenſible of it, as (in the proſecution of this work) I had frequent occaſion to peruſe a great variety of the beſt ſyſtems: in none of which (though the order of the heads is different in moſt of them) was there ſo much as an alphabetical index, which is the readieſt means (next to the dictionary form) of turning to what one wants. Many of them, indeed, have tables of contents: but, then, it is a little diſpleaſing, on any occaſion, to hunt, firſt at the begining, or end of a book, or (it may be) after the preface, for the table; and (then) through the table, for the particular that is wanted; and from thence to go to the page, when that is indicated: for, in ſome (as in Fiſher, &c.) there are no references at all. Some, in lieu of ſuch tables, give us the contents in the runing titles: where Mr Weſton wiſely advertiſes us, at the head of every page, that the treatiſe we are peruſing is ‘Of arithmetic;’ Mr Dilworth, that his is ‘The ſchool-maſter's aſſiſtant,’ &c: But, then (in this caſe) it is ſtill more tedious to turn-over the leaves from begining to end to come-at any head you would conſult; eſpecially as your labor is not without danger of a diſappointment, after-all; ſince moſt of the writers (which further encreaſes the trouble) are deficient, not only in reſpect to many particulars under moſt heads; but even in the heads themſelves. Mr Fisher (whom I mention as one of the lateſt writers) in his conciſe arithmetic (of 312 pages) has nothing at all under the heads of Alligation, Annuities, Combination, Equation, Evolution, Falſe; and Mr Weston (from whom more might have been expected, conſidering, at leaſt, the bulk of his book) has omited the doctrin of Annuities, Barter, Combination, Gain, Exchange, Intereſt, Rebate, and Tare.—But it is, it ſeems, a general perſuaſion among philoſophers, that the dictionary-way is improper for ſuch a purpoſe. ‘ When an art, or ſcience, (ſay they) ‘ is to be explaind: to break it into parcels; and re- ‘ fer to diſtant articles; is (by no means) tolerable. ‘ It gives a great deal of trouble, and perplexity; if ‘ not deſtroys that connexion and dependance, which ‘ ought to appear in all the parts of a ſcience.’ Nor is this ſayd without a ſhow of probability; ſince, by being traind-up in the ſyſtem-way (which has always univerſally prevaild) we are naturally drawn into ſtrong prejudices againſt any other; and very eaſily ſlide into this way of thinking. But, I am apt to believe, it will be found, on due examination, that the dictionary-form (if rightly conducted) does not at all differ from that of the ſyſtem, except that it has the advantage of a readier recourſe. For, ſuppoſe a perſon, unacquainted with arithmetic, going-about to learn it: The ſkeleton of the ſcience (whether in ſyſtem, or dictionary) will be the ſame [a]. Now, What will be the difference, to the learner, whether he begin with the firſt page in the ſyſtem, or the leading article in the dictionary? where he has the compaſs of his work, and the courſe of his procedure [b]. Why (1) in the

Syſtem

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[a] *For inſtance:* ARITHMETIC——*its Parts:* Notation, Numeration, Operations, Rules.———*its Operations:* Addition, Subtraction, Multiplication, Diviſion.——*its Rules:* Reduction, Proportion, Practiſe, &c.

[b] *The order*, in which every article is to be learnt, is (here) ſpecified in two verſes (line 7, 8) of the firſt page: though few of thoſe, that proceed in the ſyſtematic way, can hardly point-out the order of all, even after they have learnt them.———It may not, however, be amiſs, in this place, to obſerve, with reſpect to the doctrin of *fractions* and *evolution*; that, though, in the natural gradation of the branches of the art, they are to be learnt, as operations

after

Syſtem (which muſt be divided into chapters, or ſections, or ſomething equivalent) he muſt turn to a table of contents, or an index (and, from thence to the page) for whatever particular he has a mind to inform himſelf about: Whereas (2) in the Dictionary (which is divided into ſo many ſeparate articles) half his trouble will be ſavd, by going directly to it. . 8

The Conciſeneſs, with which the whole is drawn-up, will (I doubt not) be an objection with many; who will be apt to ſurmiſe, that, being ſo ſhort (if complete) it muſt be obſcure. But, I flatter myſelf, if they will but have patience to peruſe it with proper attention, they will find in it the double benefit of brevity, and perſpicuity [c].—I ſhould be *loth* .. to oppreſs

and

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after the 4 fundamental ones, to which they are referable (V. p. 4, note [f]) yet it will, perhaps, be better to let them alone, till, in the courſe of proceeding through the rules, they ſhall be found neceſſary for the ſolution of any queſtion in any of them: the rather, as the much greateſt part of ſuch calculations, as occur in the ordinary occaſions of buſineſs, may very well be managd without the knowlege of them.

[c] The more ſo, as *the rules are detacht from the exemplifications, and diſtinguiſht by a larger letter.* Hereby the learner will know what he is to commit to memory, in order to have a clear conception of the whole ſcience. And, by this means, his practiſe being always directed by the theory he has treaſurd-up in his mind, he will be leſs liable to forget; and may more eaſily recover what may have ſlipt, through diſuſe; than can poſſibly be expected from other ſyſtems: in all which there is ſuch a huddle of inſtructions (always undiſtinguiſht, and oftentimes verboſe) that both the eye and the mind are at a loſs what to pitch-upon for the fixing of thoſe ideas, which are found, of all others, to be the moſt fleeting. Beſide that, a multiplicity of words, where a few would anſwer the purpoſe, is apt to confound the mind, as well as to check the ſpirit in the purſuit of knowlege.—Dr WELLS (in his Young gentleman's arithmetic

and confound a learner with 34 pages of tedious inſtructions, to acquaint him with a couple of troubleſom methods of Diviſion; as Mr Fisher has done (arithmetic, in the moſt conciſe method, p. 89—123) and thinks he has done bravely too (pref. p. 3) when a much ſightlier, ſhorter, and eaſier way (with the ſeveral compendiums of it) might be taught him, in 7 hexameter verſes (readily applicable, and hardly to be forgoten) with about 2 pages of exemplifications, &c.. or to ſwell the doctrin of Practiſe into a great many long rules, through 37 pages, which he thinks (preface, and contents) he has done with greater variety and brevity than has been done in any one book extant: when a full account of it may be given within the compaſs of 3 lines of rules, and about 2 pages of exemplification; and that, too, with greater evidence to the underſtanding, and far greater ſecurity to the memory.——I ſhould be *aſham'd* alſo to offer a com-

pendium

metic) thought it neceſſary to ſpend above 8 pages in octavo, to make plain the doctrin of numeration; when the whole of what he has ſayd thereon may be pictur'd-out, and made perfectly intelligible even to a child, within the compaſs of 2 verſes explicatory of a table of half a line. To what purpoſe, then, ſuch a profuſion and cumber of words; that do ſo little honor to a note of his (p. 9) wherein he is pleaſd to declare, notwithſtanding the length of his inſtructions, that, It can be experimentally ſayd, that ſeveral young gentlemen have (by the method there explaind) been taught, in a very few minutes time, to read ſums of 20, 40, 60, figures.'——The Analyst (in his Introduction to the mathematics, 1746) with a grand apparatus of definitions, illuſtrations, corollaries, problems, theories, ſcholiums, &c. has ſpun-out his doctrin of fractions to 85 pages: whereas, without all that pomp and parade, a fuller account of them, and with much greater evidence, is (here) exhibited, in one view, in 26 verſes; with an illuſtration thereof, by a great variety of examples, in 4 pages.

pendium of arithmetic (both practical, and theoretical) as Mr. DILWORTH has done; without particular directions, for working the operations; the omiſſion of which he gives (ſchool-maſter's aſſiſtant, preface, p. 2) as a reaſon for his book's being reduc'd to the narrow compaſs (of 168 pages) it appears in. In effect, the only value of his performance is (what the general title imports,) that it may ſerve to take-off the heavy burthen of writing-out queſtions, which teachers have long labord-under: inaſmuch as he has given us a larger liſt of queſtions than we generally meet-with in books of this nature. With the ſame view, therefore, I have drawn-up an Appendix to my ſyſtem (and which may equally ſerve as a ſupplement to any other) in which there will be found ſeveral peculiarities of great uſe to the eaſing of the maſter's trouble: beſide a ſeries of exerciſes (without the interruption of other matters) that will open the mind of the learner to all the varieties of views, that either buſineſs, or curioſity may require. 9

The Title, how paradoxical and preſumptuous ſoever it may appear, will (I hope) be excus'd; if the work ſhall be found anſwerable to the high pretenſes. I ſhould, indeed, have been better pleasd with the plain title of a New ſyſtem of arithmetic: but that I conſiderd, with the accurate Mr. Malcolm (arithm. pref. p. 5) that 'When a ſubject has gone through ſo
'many hands as arithmetic has done, a new book
'cannot want many prejudices againſt it: and, there-
'fore, that to ſend it into the world without ſome in-
'troductory account of it, is no better than laying it
'down at random, or (more properly) expoſing it.
''Tis an unreaſonable neglect of ſomething that
'equally concerns the author, and the world. For, if
'an author has endeavord to do ſomething more uſe-
'ful and complete on any ſubject than has been al-
'ready done, and thinks he has (in ſome meaſure)

‘ ſucceeded: as the telling the world ſo, may be done
‘ without any breach of modeſty; ſo it appears equally
‘ juſt and neceſſary to explain particularly wherein the
‘ improvements and advantages of the work lie: that
‘ every one may ſee how far it anſwers their purpoſe,
‘ and deſerves their encouragement. It is true, it
‘ muſt ſtand upon its own baſis: yet nothing ſeems
‘ more honeſt and reaſonable than this kind of invitation
‘ to look into it. It may be objected, I know,
‘ that Here is only the author's word for this account;
‘ which is a partial teſtimony: but, if it be conſiderd
‘ that he ventures his credit, as well as the ſucceſs
‘ of his work upon a fair repreſentation: this, it
‘ may reaſonably be hop'd, will incline the more candid
‘ and charitable to believe that it is ſo.’ 10

As to the Freedoms I have uſd in finding fault with others, I take the liberty to declare—in the firſt place, that I ſhall be ſo far from taking-amiſs the like uſage from others; that I ſhall look-upon it as a kindneſs done to my-ſelf as well as to the public, to be convicted of any miſtakes I may have been guilty-of, or to be ſhown wherein I have fallen-ſhort of what I profeſs to have aimd-at—and, in the next place (1) I think I have the ſame apology (and a good one too) for what I have done here, as I thought neceſſary to make on another occaſion (in a critique on Lilye's, and the Weſtminſter grammars) viz. that, in an attempt at any thing extraordinary, it is impoſſible, however neceſſary it may be to attain the end, either to point-out the excellencies one aims-at, without an air of vanity; or to ſhow another his faults, without the appearance of ill-natur'd rudeneſs, or inſolent civility. (2) Not to inſiſt on the authority of the beſt of writers on this ſubject, Mr. Malcolm; who has criticizd two fam'd arithmeticians; and that, as he tells us, (arithm. pref. p. 7) in his own vindication; to-wit, that he might not ſeem to offer to the world a work, that muſt have been deemd impertinent, or ſuperfluous; if Mr. Hatton's book had anſwerd the title; or Mr. Hill's, the recommendation given it by ſo good a judge as Mr. Ditton. (3) And I will venture to add (without offenſe, I hope, to that excellent arithmetician) that I have renderd the whole of the ſcience much fiter for the uſe of ſchools than his ſyſtem was deſignd-for; and, in many things, made conſiderable improvements, and additions; which (I dare believe) he will be pleaſd-with: at the ſame time that I have omited nothing that he has given us, except demonſtrations, and ſuch particulars as (more properly) belong to algebra: which, though within the compaſs of his deſign, were foreign to my purpoſe and propoſals. 11

The Frontiſpiece, which was deſignd more for uſe than ornament, is all (I think) that remains to be conſiderd. In order to the explication of it, the reader will be pleaſd to obſerve that it *contains the rudiments* of the art; with which if the learner be perfectly well acquainted [d] (ſo as to ſay them without ſtudy, doubt, or heſitation) the whole buſineſs of operation will be eaſy, and expeditious.—The *firſt* table gives the figures of the arabic characters, in the three forms of integers, decimals, and vulgar fractions, with the arithmetical ſymbols or ſigns of the operations, &c. the explanation whereof may be ſeen in the ſyſtem, line 9: being here preſented, only to ſtrike the imagination with the form and power of each.—The *ſecond* table is alſo deſignd as a picture, to ſhow the manner of reading any ſeries of thoſe figures, according to the explanation given thereof, line 4, 5, 6.—The *third* table exhibits the firſt principles of addition and ſubtraction. In learning it (I) To Add, ſay (1) either 6 (to the left) and 3 (at the top) is 9, where the rows meet (2) or, reverſely, 3+6=9; and ſo on [e]. (I) To Subtract, ſay (1) ei-

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[d] *To be perfectly well atquainted with theſe rudiments*, the only means, is (1) Firſt, to form, and fix a ſenſible idea of the powers and combinations of numbers, by the uſe of counters *, or ſome ſuch palpable objects; which a child will tell-out with pleaſure, according to the ſeveral diſpoſitions of them, to which he ſhall be directed; for the better anſwering the various purpoſes of the fundamental notices. (2) And, then, to imprint them on the mind by frequent repetition.——It may not be improper, therefore, to accuſtom children to them very early; that the ſounds may grow familiar, before they begin to reflect on the reaſons of them. For, if they offer to caſt-about in their thoughts to find-out, what they ought to know by rote and without reflexion; they cannot be ſaid to be perfect enough in the uſe of the rudimental tables, to be able to proceed to the rules with pleaſure.———This is, indeed, the reverſe of what I adviſe

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adviſe on all other occaſions; not even approving a child's geting ſo much as his catechiſm, without firſt giving him (by familiar explications, queſtions, and canvaſings) ſo much underſtanding of it as his tender years will admit, and the nature of the thing will allow. But, here, the caſe ſeems to be quite different: becauſe nothing more is deſignd, hereby, than the fixing in the memory a concatenation of ſounds, without any regard had to their uſe or application. If therefore a perſon, in multiplying (for inſtance) 7 into 6, muſt fall a-reaſoning that 3 times 7 is 21, and that twice 21 is 42, &c. he will find himſelf ſadly perplext, and ſlow in his operation: but if 6 times 7 draws-in the ſound of 42 (and ſo of the reſt) as readily as twice 2 does that of 4; every thing will be eaſy, without any effort, or embaraſment of the mind.

* *Dr Record* (arithm. p. 179—216) has been very particular in his inſtructions for accounting by counters: 'which 'feat, ſays he, doth not only ſerve for them, that cannot 'write and read; but alſo for them, that can do both; but 'have not, at ſome time, their pen, or tables ready with them.

[e] N.B. The digits being known, Every thing beyond will be eaſy. For——The *decads* (10, 20, &c.) with any digit

ther 3 (to the left) from [f] 9 (where the rows meet) and there remains 6 (at the top) (2) or, reverſely, $6-9=3$; and ſo on [g].—The *fourth* table carries the doctrin of multiplication and diviſion as far as is neceſſary to prepare for the performance of the operations in all caſes [h]. In learning it (I) To Multiply, ſay (1) either 3- (to the left) -times 6 (at the top) is 18, where the rows meet; (2) or, reverſely,

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git, are known by the name: Thus, 10 and 8 is 18; $20+8=28$; &c.—Any *beyond the decads* (as 13, 23, &c.) are known by reflecting on the digits. Thus, 13 and 9 (by thinking that 9 and 3 is 12) appears to be 22; and $23+9$ (by the ſame conſideration) $=32$, &c.

[f] NB. Though I (here) direct the performance of ſubtraction according to the cuſtomary way; I am apt to think that the doing it, addition-wiſe, is, on many accounts, the moſt eaſy, and the moſt commodious. V. Subtraction, note [b]. Diviſion, note [d]. See, alſo, the next Note [g].

[g] From this explication of this table it appears that *Subtraction is the reverſe of addition:* and, indeed, if we ſtudy to find-out an anſwer to a queſtion in the former, it is (chiefly) by attention to the latter, that we come-at-it. Thus, to take 3 from 9, we conſider how many more than 3 make 9; and ſo hit-upon 6. In like manner to take 6 from 19 we think that 6 and 3 make 9; and conſequently 6 and 13 is 19. So, we ſay 6 from 19, and there remains 13. V. note [f].

[h] From this explication of this table it appears that *Diviſion is the reverſe of multiplication:* and, accordingly, to find, How-oft one number is containd in another, we multiply it (in our minds) from ſome lower ſtep, till we come ſo near, as that the next would carry us beyond it. Thus, ſuppoſe I enquire how oft 9 in 71. I begin (by gueſſing) at 4 times 9, which is 36; and go-on $5\times9=45$; $6\times9=54$; $7\times9=63$; $8\times9=72$: by which I find that the anſwer will be 7 times, and 8 over. V. Diviſion.

verſely, 6×3=18[i]; and ſo on[k]. (II) To Divide, ſay (1) either, How oft 3 (to the left) in 18 (where the rows meet) anſwer, 6 at the top; (2) or, reverſely, 6÷18=3; and ſo on.——The *fifth* table, which *is a key to the mnemonic art* (and may be got-by-heart in a few minutes) needs no explanation: only, . . . For the readier geting, and remembering the numeral force of the letters, it may be obſervd that (I) the Cipher is denoted by the laſt vowel, and conſonant. (II) The Digits (1) in the Upper line (1 2 3 4 5) by the vowels in their order: 6 7 9, by the diphthongs, according to the value of the vowels added together: 8, by the initial letters. (2) In the Under line: 1, by the firſt conſonant; 3 4 6 9, by the initials; 2 7 8, by their alluſion to Duo, ſePtem, οKτω: 5 to L; which (in roman notation) ſtands for 50. (NB) To diſtinguiſh the ſound of *y* from *i*, it may be pronounc'd *wee* ſhort. 12

The Liſt of Authors, at the end of the work, though of little uſe to many, is a curioſity, that ſome will not be diſpleasd with. I have (always) been of opinion, that, Before a man undertakes to write a ſyſtem of any ſcience, he ought to conſider what has already been done: leſt he ſhould loſe his labor, and expoſe himſelf too, by offering to the world what (for ought he knows) may have been done much better: not to mention the advantages he might make both of the errors and excellencies he may diſcover in the performances of thoſe, whom he endeavors to excel. I cannot indeed ſay that I have peruſd all the authors in this liſt: but preſume I have reaſon to believe there will nothing of moment be found in any of them, which I have not here given with ſome improvements; inaſmuch as I have carefully conſulted the moſt conſiderable of the ancients, and thoroughly ſtudied almoſt all the moderns. If I am miſtaken in any particular, I reckon I have given provocation enough, by my pretenſes, to be told of it; and ſhall think my-ſelf oblig'd to any one that will be ſo good as to do it. I ſhould be glad, alſo, if the curious would be pleaſd to communicate the names of ſuch authors as I have not been able to recollect; or that never came under my obſervation; and (for that reaſon) are not to be found in this catalogue. 13

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[i] Though, poſſibly, it may ſuffice to get the table in the ſhort way, by begining each digit to the left with the ſame at the top, e. g. 6×6, 7×7, &c.——Leſs perplexity will ariſe hence: and, after this way is well known, a child will eaſily be apprizd of the reverſe way.

[k] NB.——10 is (here) *omited:* becauſe the anſwer is imported by the name: 2 tens is twenty; 3×10, thirty, &c. ——11 is *omited:* becauſe it is only puting the multiplier before itſelf. Thus 4 times 11 is 44, 5×11=55, &c.——12 is *added*; becauſe the ready knowlege of it is very convenient as a pence-table; and for ſundry particulars in reduction.

14 NO 63

* * *

ARITHMETIC
THE
DOCTRIN of NUMBER.

Parts of Arithmetic:

1 NOTATION, NUMERATION, RULES, OPERATIONS.

2 NOTATION: Doctrin of *Signs* [a]; and of *Figures* [b]: *digits* (nine) and a *cipher* [b].

3 NUMER: Expreſſion of *Numbers: fractions,* Parts; *integers,* Wholes note [c].

4 *Ten* of the Row-to-the-Right make *One* of the Row-to-the-Left [d] — And

5 *Period,* ſix Figures: By *threes* write, and read; to-wit, Units (1) Tens (x) Hundreds (c).

6 *Commas* note Thouſands; *Dots,* Periods: 2d, Millions; 3d, Trillions; and ſo on [e].

7 OPER: Addition, Subſtraction, Multiplication, Diviſion [f]. - - - - - - - - - 4

8 RULES: Re. Pro. Prác. In. Reb. Ex. Tare: Fél. Ba. Gain: Cómb. Al. Equ. Ev. Falſe [g]. - - - - - - - - - - - - 15

in handling of which, regard is to be had to

9 *Signs:* Add, *and* (+) Sub, *from* (—) Múlt, *into* (x) Div, *by* (÷) E'qual-to (=) So is (::) [h].

10 *Denominations* (different) of Coins, Meaſures, Weights: Vide *Tables.*

A ABBREVIATURES *Explaind:* Alligation. Barter. Combination. Equation. Evolution. Exchange. Fellowſhip. Intereſt. Practiſe. Proportion. Rebate. Reduction.

E EQUIVALENTS *Referd:* Alternations = *combination.* Arbitration, *exchange.* Commutation, *barter.* Company, *fellowſhip.* Compoſition, *combination.* Diſcount, *rebate.* Elections, *combination.* Extraction, *evolution.* Factorſhip, *fellowſhip.* Loſs, *gain.* Partnerſhip, *fellowſhip.* Permutation, *combination.* Poſition, *falſe.* Profit, *gain.* Suppoſition, *falſe.* Trucking, *barter.*

a For the *Signs*, or Characters, expreſſive of the ſeveral operations, *&c.* V. line 9.

b The *Figures*, or Arabic characters, are—nine *digits*, (1 one, 2 two, 3 three, 4 four, 5 five, 6 ſix, 7 ſeven, 8 eight, 9 nine) and — a *cipher*, or Nought, or Ought (as it is vulgarly calld) to-wit 0: for the Uſe of which, See note [d], and [c] †. See alſo Notation-Table, in the Frontiſpiece.

c *Fractions* (importing the Parts, or Subdiviſions of an Integer) are either Vulgar, or Decimal——To give an Idea of both: *Four, and three fourths* are expreſt (1) in *vulgar* Fractions, thus: $4\frac{3}{4}$ * (2) in *decimal* Fractions (a Dot, or Comma, being the Separatrix; which, in Mixt numbers, diſtinguiſhes the Integer from the Fraction) thus: 4.75, or 4,75 †.

* $4\frac{3}{4}$ is calld a *Mixt* Number, as conſiſting of——4, an *integer*; ſignifying ſo many Wholes, for inſtance, Shillings: & ——$\frac{3}{4}$, a *fraction* (of which, 3, the Upper, is calld the Numerator; and 4, the Under, the Denominator) importing 3 Parts of a Whole divided into 4; that is, Three-Fourths, or Three-Quarters. Accordingly, $4\frac{3}{4}$*s* is 4*s* 9*d*; 1 fourth of a ſhilling being 3*d*; and, conſequently, 3 fourths, 9*d*.

† 4.75 is equivalent to $4\frac{3}{4}$: the denominator of .75 being conceivd to be 100; which bears the ſame proportion to 75, as 4 does to 3: for, 4 : 3 :: 100 : 75.—— In like manner, .5 is $\frac{5}{10}$ (that is, 5 Tenths, or 1 Half) .25 is $\frac{25}{100}$ (that is, 25 Hundredths, or 1 Quarter) and, ſo, in all caſes; taking the figures expreſt for a numerator; and ſuppoſing 1 under the ſeparatrix, and ciphers under the figures, for a denominator——So that a decimal is a fraction, whoſe denominator (not expreſt) is to be underſtood to be 10, or ſome power of 10; viz. 100, 1000,

1000, &c.——And hence it appears that ciphers, at the end of a decimal fraction, have no value: .5 being equal to .50, or .500; ſince 5 bears the ſame proportion to 10, as 50 to 100, or 500 to 5000. V. note [d] *.

d Thus (in the Numeration-table, in the frontiſpiece) ten of the 1ſt row (to the right) make one of the 2d; to-wit one ten (expreſt by 1 before a cipher, thus: 10): and ten of the 2d row make one of the 3d; that is, ten tens make 1 hundred; to expreſs which 2 ciphers are to be plac'd after the 1, thus: 100; and ſo of the reſt——— And hence appears the uſe of the *cipher*; viz. to raiſe the value (by altering the place) of the figures to the left of them *: Thus 9, by one cipher added, is removed to the place of tens; and becomes 90 (that is, 9 tens, or ninety) by two, 900 (nine-hundred) &c.——And, from this conſideration, we find that a preciſe idea may be formed of any number, be it ever ſo great, and by what name ſoever calld; ſo we do but know its diſtance from Unity. V. note [e].

* Before figures (to the right of them) they have no value [05, 005, are but 5] except in decimals, where they diminiſh [.5 is $\frac{5}{10}$ or $\frac{1}{2}$, .05 is $\frac{5}{100}$, .005 is $\frac{5}{1000}$ &c.] And it is to be obſervd that the diminution encreaſes in proportion to the riſing of the numbers: Thus the denominative value 'Hundreth' is leſſer than that of 'Tenth'; foraſmuch as the Hundreth part of a thing is ten times leſs than the Tenth of it. V. note [c] †.

c To initiate a child in *reading* by means of the Numeration-table in the frontiſpiece——Let him begin to practiſe on the three figures of the ſemiperiod to the right; and fix an idea of the value of their places by the terms of units, tens, hundreds. So 21 will appear to be 2 tens (or twenty) and 1 unit; to be read twenty-one: and 321 (to-wit, 3 hundreds, and 2 tens, and 1) will be three hundred and twenty-one.——Then, regard had to the names of the periods, he may (in the ſame manner, and with the ſame eaſe) *read* the longeſt ſeries of figures. Thus the example, in the frontiſpiece, will be a hundred twenty three thouſand (viz. trillions; but not to be expreſt till you come to the end of the period) 456 Trillions *: 789 thouſand, 987 Millions: 654 thouſand, 321.——And, in this manner, any ſum, how long ſoever, being commad and dotted, is (as it were) ſpelt; and becomes as legible as a ſyllable of three letters——And, which adds greatly to the conveniency, the longeſt ſum may be *writ* forward, without the leaſt heſitation, or retroſpection from the laſt figures: ſo that there is no need of ſaying, through a long train of figures (as is uſual) 'units, tens, hundreds, thouſands, tens of thouſands, hundreds of thouſands, millions, &c.' †—— A tyro may, indeed, be puzzled by a different way of expreſſion; but may eaſily be led into a right conception. For example, To write-down 11 thouſand, 11 hundred, and 11; he may be apt to ſet-out wrong: but, give him the hint that 11 hundred (by virtue of the comma) is 1,100 [that is, 1 thouſand, 1 hundred] and he will add that thouſand to 11,000; which will make the number (in plain numeration) 12,111; as will appear more expreſsly by addition, as in the margin. By like conſiderations he will find that ſix ſcore and 10, is 130; three dozen and 4, is 40; &c. V. Frontiſp. & pref. 11.

$$\begin{array}{r} 11{,}000 \\ 1{,}100 \\ 11 \\ \hline 12{,}111 \end{array}$$

* *This period*, by cuſtom time-out-of-mind, is calld, even to this day, *millions of millions*; the 4th, of courſe, millions of millions of millions; and ſo on: but the terms are too vague and embaraſſing, to deſerve regard.——By the modern reformers it is commonly calld *billions*: but may, I think, be more inſtructively calld *Trillions*, from tres three; as being the third, in order. And, agreeably hereto, the riſing periods may be denominated, from their relation to the firſt, Quatrillions, Quinquillions, &c.

† Thus, To write *forty millions, two hundred thouſand and eight*: attending only to the names and powers of the periods, I ſet-down 40. (with a dot, to import the period; to-wit, millions) Then 200, (with a comma, to denote the ſemiperiod of thouſands) And, with regard to the laſt ſemiperiod, having neither hundreds, nor tens; I fill-up the places with ciphers, and write 008. So, it is 40.200,008.

f *The Operations* of arithmetic conſiſt in the managing and working of figures, in order to the ſolving of queſtions.——They are, properly ſpeaking, only the four mentiond in the text; the doctrin of *evolution*, & *fractions* being referable to them.——For the *proof* of them, or the means of being ſatisfied that they are rightly performd; V. Proof.

g *The Rules* of arithmetic, as they are commonly calld, give particular directions how to apply the operations for the ſolution of queſtions—— *For the order in which they are to be learnt*; that, in which they are here recited, may (perhaps) be the moſt commodious. However, ſince, as Mr Dilworth (pref. p. 7.) obſerves, there are hardly two maſters, that follow the order of the rules alike; ſome liking beſt to teach that rule firſt, which another thinks convenient to teach afterward; while another looks upon it as a matter quite indifferent, among ſome rules, which he teaches firſt: it is to be hop'd, no objection will be made, either to the order I here propoſe; or to that of the book, which is diſpos'd alphabeticaly; for, notwithſtanding this difference in the diſpoſition of them, every body may here (moſt readily) turn to that rule firſt, which he likes ſhould be taught firſt. V. Pref. paragra. 7.

h *The Signs*, or characters, made-uſe-of by arithmeticians, more conciſely and commodiouſly to denote the ſeveral operations, &c. are as in the text, and frontiſpiece. —*For example* (1) $2 + 4 = 6$ (2) $9 - 6 = 3$ (3) $3 \times 6 = 18$ (4) $12 \div 3 = 4$ (5) $3 : 6 :: 4 : 8$. ——*Read thus:* (1) 2 and 4 is 6 * (2) 6 from 9 is 3 † (3) 3 into 6 is 18 ‡ (4) 12 by 3 is 4 § (5) as 3 is to 6, ſo is 4 to 8 ‖. V. Frontiſp. & pref. 11.

* Or, 2 plus [more] 4 is equal to 6.
† Or, 9 minus [leſs] 6, there remains [reſts] 3.
‡ Or, 3 times 6 makes 18.
§ Or, 12, by 3, gives [quotes] 4. Or, the threes in 12, 4. §
‖ Or, If 3 gives 6, 4 will give 8 ‖.

Diviſion is alſo expreſt (1) by reverſe Parentheſes : 3) 12 (4, (2) by numbers plac'd Fraction-wiſe: $\frac{12}{3}$; the lower number being the diviſor; the upper the dividend.

ADDITION.

1 ADD. Place the figures in rows: units
under units; to tens, tens; &c.
2 (of different námes, ſame to ſáme; in each,
leſs than th' amounts to the next name)
3 Caſt-up the laſt, and the total ſet-down:
Thén, go backwards [a]——In *Deep* rows

4 (I) Dot the Amounts-to-the-next-name *: thither carry 'em, having writ the Remainder [b].

5 (II) Or, by Diviſion, the amounts find † (1) when Eaſy (2) or book to be kept Fair [c].

6 *Shillings* as Integers: half even-tens carry; ſet down the odd one [d].

* *When it is Fiteſt to Dot* [e];

7 Clear-up the tens with the units; though added aſunder, in ſumming [f].

† *When*

The Amounts-to-the-next-name are Fractions;

8 Bring the ſum to the leaſt name: Divide by what the amount makes [g].

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a *For example:* To add 24 20 and 3 —— Having plac'd them in rows, units under units, tens under tens, as in the margin —— Caſt-up the laſt row, that of units; Saying * 3 + 4 = 7: which ſet-down directly under the row —— Then, going-backwards to the row of tens, Say 2 + 2 = 4: which ſet down likewiſe; and the ſum, or total (or, as ſome call it, the ſum-total) is 47. V. Front. & Pref. 11.

24
20
3
—
47

* Or, rather (after a little practiſe, at leaſt) *adding* the particulars, *without naming* them: which is the eaſieſt and ſpeedieſt way; ſo many ſounds as there are figures being ſpar'd, and the mind (unembarraſt with din) more free to think.

b *For example:* To add-up the ſum in the margin:—Having plac'd, in rows, denomination under denomination †, units under units, &c. Begin at the laſt row, that of farthings ‡; and ſay: 2 + 3 = 5; that is 1 penny (the amount to 1 of the next row) and 1q: ſay, therefore, Dot, and go 1.

Denomin.	*l.*	*s.*	*d:*	*q.*
Particulars.	4	18.	11.	3
	9	16	9.	2.
	*—	—	5	3.
	7	19.	10.	2
Total —	22	16	1	2

——Then, that 1 (carried-on) $+ 2 = 3 + 3 = 6$ q: that is 1 d (the 2d amount to one of the next name) and 2 q: which q ſet-down (having no more to add in that row) and carry the 2 dots, or 2 d, to the next row——And ſay: 2 (that I carry) $+ 10 = 12$ (the firſt amount to 1 of the next row) Dot, therefore, and go-on, as in the farthing-row, with the pence and ſhillings.

NB. The Firſt row (in all caſes) is conſiderd as conſiſting of *integers*; and is, therefore, in regard to the decuple proportion of the rows (mentiond under the article Arithmetic, line 4) to be caſt-up, without doting, by carrying the tens (that each row amounts to) to the next; and ſeting-down the overplus. V. note [d].

* *A daſh*, inſtead of a cipher, is more readily made; and leaves the ſum opener to the eye, and eaſier to be caſt-up.——*Formerly*, two ciphers were uſd (under a notion of uniformity) to ſet-down nothing.——And, *even Now*, it is a very common practiſe to make two ſlope-daſhes between every denomination; which is taking pains, to ill purpoſe.

† *The particular denominations being reduc'd to the uſual ones, if expreſt otherwiſe.* For example, Six and thirty ſhillings = 1 l 16 s: Nineteen pence = 1 s 7 d: Three half-pence = 1 d. 2 q.——And, thus, in receiving mony of ſeveral coins; each ſpecies being eſtimated aſunder, the total will ariſe more eaſily. Thus, ſuppoſe I were paid a ſum in 3 moidores, 2 portugal pieces, 5 guineas: I ſay (for the moidores) 3 × 27 s = 81 s; and ſet-down 4 l 1 s: Then (for the portugal pieces) 2 × 3 l = 6 l; and 2 × 12 s = 24 s; in all 7 l 4 s. Then (for the guineas) 5 × 21 s. = 105 s; that is 5 l 5 s. Then 4 l 1 s + 7 l 4 s + 5 l 5 s = £ 16 10: To which may be added whatever ſilver, &c. is paid with the aforeſaid pieces. V. Reduction, note [b].

‡ *Farthings* are commonly ſet-down fractionwiſe ($\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$) which Mr Fiſher (p. 14) will have to be the beſt way; though it is hard to ſay on what account; there being three ſtrokes of the pen inſtead of one; beſide the unſightlineſs to the eye, and embaraſment in the operation, without any one advantage.

c *For example*: In the above-ſum (note [b])——Caſting-up the whole row, ſay $2 + 3 = 5 + 2 = 7 + 3 = 10$ q: or rather, looking on the figures (without pronouncing them) ſay 5 7 10; according to the hint, given note [a].——Then, the number of farthings in a penny (the next name) being 4; ſay, the fours in ten: Anſw. twice; which is 2 d (to be carried to the next row) and 2 q over; to be ſet-down: And ſo on.——*NB.* Thus (in lieu of pence-tables, &c. [*]) (I) *A diviſor* (1) when ſmall enough for the head, gives the amounts to-the-next-row (2) and when too-large, may be workt on a ſlate. (II) Or (till diviſion is learnt) the amounts, as they ariſe may be noted (1) by clenching the fingers ſucceſſively, (2) or by doting on a bit of paper applied to the edge of the row; in order to keep the account fair, and unblur'd. V. note [e].

[*] *Pence-tables* [20d is 1s 8d: 30d is 2s 6d, &c.] are (however generaly recommended) not only unneceſſary, but embarraſſing. Setting-aſide the charging of the memory with ſuch tables (the uſe of which extends no farther than to pence) it is eaſier, if I would know the value of 87d, to ſay 12 in 87: anſwer 7, and 3 over, (to-wit, 7s 3d) than, firſt to conſider that 80d is 6s 8d; and then add 7d: which, a ſudden thought does not ſo readily hit-upon to be 7s 3d as in the aforeſaid manner, wherein is requird no more attention than to the multiplication-table, which muſt be known.

[d] *Shillings* are moſt commodiouſly caſt-up as integers (V. note [b]) in which you carry the tens of the preceding row to the next; and ſet-down the overplus——Thus, in the ſhilling-row (note [b]) 3 (that you carry) + 9 + 6 + 8 = 26: ſet-down the 6 (the overplus of the tens) and carry the 2 tens to the next row; where 2 (that you carry) + 1 + 1 + 1 = 5. Then, having ſet-down the odd-one, carry to the next row (that of pounds) the half of the even tens, to-wit, 4: that is, 4 ten ſhillings; or 2 pounds.

[e] *That is,* I think, when the amount-to-the-next-name is too great to divide-by with the head only: though Mr Malcolm (arithm. p. 81) is of opinion that there is leſs trouble in doting; and, even, that it is more convenient than to do the work by diviſion (when one can do it ſo) becauſe more ſimple. V. note [c].

f *For example*: In caſting-up the ſum in the margin: ſay——Firſt, 41 + 7 = 48 + 30 = 78: dot (at 60, the number of minutes that make an hour) and carry 18——Then, 18 + 9 = 27 + 40 = 67: dot, and go 7. ——Then, 7 + 6 = 13 + 50 = 63: dot, and ſet-down (the overplus) 3.——Then, carrying the dots to the next name, proceed as in integers. V. note [b].

hrs	min.
23	56.
22	49.
16	37.
19	41
83	3

g *For example:* In caſting-up the ſum in the margin, Say —— For the *runlets* (1) Firſt 1 + 1 + 1 + 1 = 4 runlets (2) Then (there being $1\frac{3}{4}$ of them in a barrel) bring them to the leaſt name, *viz.* quarters; ſaying 4 × 4 = 16. (3) Then, 16 ÷ 7 (the number of quarters, that make the amount-to-the-next-name; to-wit, barrels) = 2 barrels (to be carried) and 2 quarters, or half a runlet to be ſet-down)——For the *barrels* (1) Firſt 2 (that you carry) + 1 + 1 + 1 + 1 = 6 (2) Then (there being $1\frac{1}{3}$ of them in a tierce) bring them to the leaſt name, to-wit thirds; ſaying 3 × 6 = 18. (3) Then 18 ÷ 4 (the number of thirds, that make the amount-to-the-next-name; to-wit, tierce) = 4 tierce (to be carried) and $\frac{2}{3}$ (to be ſet-down).—— For the *tierce*, proceeding in the ſame manner; and——For the *hogſheads*, caſting-them-up as integers (V. note [b]) The total will be as in the margin.

hogſh.	tierc.	barr.	runl.
24	1	1	1
32	1	1	1
59	1	1	1
19	1	1	1
138	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{1}{2}$

ALLIGATION.

1 ,AL. for propórtioning ŏf míxtures * is Médial (l. 2.) or Alternate (l. 3.)

2 M'EAN *rate of míxtures to find:* As the Thíngs to their Rátes; ſo a Part is [a].

3 M'IXTURES *to máke for a Ráte* [b]: (1) Counterplace the dífferences ŏ'th' extremes; [c].

linking

4 línking a gréat and a léſs[d]; and an ódd-one to éach of the others, [e].

5 (2) *Pártial:* As dífference of gíven to gíven; ſo ŏf ſoúght unto ſought is [f].

6 (3) *Tótal:* As dífferences ŏf quántities; ſó to its quántity each difference [g].

* *Hither may be Referd the Doctrin of*
ALLOY.

7 Alloy: the mixture of copper, that makes Gold and Silver *ſtandard*, &c. [h].

8 *Gold:* carat ed, copper e [i]. *Silver:* penny-weight ded, copper aei [k].

9 V'alue [l]: *Ounce*; Cópper, u,trŏ farthings [m]: Sílver, auz pence [n]: Gold, o pounds [o].

10 Therefore, *Grains*; Guinea, ben: Shillin, oui: Half-penny, *Drams* i:

11 And, the want of *a Grain*, in Gold, 2-pence; Silver $\frac{1}{2}$-farthing [p].

[a] *For example:*—I. A *merchant* has 13 gallons of wine at 17s per gallon; 11 gallons, at 15s; and 19 gallons, at 14s. If theſe are mixt, What is the price of 1 gallon of the mixture? (Anſw.) As 43 g to 652s (the value of the gallons, according to the rate *) So 1 gallon to 15s 1 d $3\frac{35}{43}$ q. (Note) If we ſuppoſe 6 gallons of water (whoſe value is nothing) mixt with theſe; the proportion is this: If 49 g coſt 652s; what 1 gallon?——II. A *goldſmith* mixes 7 ounces of gold, 23 carats fine, with 13 ounces 19 carats fine: What is the quality of the mixture †? Say, As 20 oz to 408 carats (of pure gold: the value, in quality, of the 20 oz †) So is 1 oz (of the mixture) to $20\frac{2}{5}$ carats.

* For—If 1 g gives 17*s*; 13 g will give 221 *s*.—And, if 1 g gives 15*s*; 11 g will give 165*s*--And, if 1 g gives 14*s*; 19 g will give 266*s*—Then, $221 + 165 + 266 = 652$ *s*.

For the nature of this mixture, V. Alloy, l. 7.

† For—If 1 oz has 23 carats (of pure gold) 7 oz have

161 carats.—And, If 1 oz. has 19 carats (of fine gold) 13 have 247 carats——Then 161 + 247 = 408.

b *Alternate* Alligation has 3 caſes; and may be calld (1) *Simple*, When the mean rate of the mixtures is given; but neither the whole quantity of the ingredients to be mixt, nor any part of them, is limited. (2) *Partial*, When one of the quantities is given, with the mean rate of the whole mixture. (3) *Total*, When the ſum of all the quantities is given, with the mean rate of that ſum.

c *For example:* How much wheat (at 5s the buſhel) and rye (at 3s 6d the buſhel) will make a mixture that may be ſold at 4s 4d the buſhel? ——*Anſw.* (the prices being brought into one name)

d. 52	60d. Wheat. 10 the differ. of the leſs extreme	counterplac'd
	42d. Rye. 8 the differ. of the greater extreme.	

So, the mixture requird will be 10 buſhels of wheat, and 8 of rye.—*Proof:* As 18 (the ſum of the buſhels, 10 + 8) to (600 + 336) 936 (d. the total of the price of 10 buſhels, at 60d; and 8 buſhels, at 42d.)—So is 1 (buſhel of the mixture) to 52 (d. the mean rate.

d *For example:* A *vintner* would make a mixture of malaga (worth 7s 6d a-gallon) with canary (at 6s 9d a-gallon) ſherry (at 5s a-gallon) and white wine (at 4s 3d a-gallon) what quantity of each muſt he take, that the mixture may be ſold for 6s, a-gallon?——Anſw.

d. 72	90	Malaga	21	or	90	Malaga	12	72 d.
	51	White	18		60	Sherry	18	
	60	Sherry	9		81	Canary	21	
	81	Canary	12		51	White	9	

—Suppoſe it were requird that the whole mixture ſhould conſiſt of 20 gallons: Say, As 60 (the number of the gallons of all the wines, found by the differences) to 21 (the gallons of malaga found by the differences) So is 20 (the number of the gallons required to be mixt) to 7 (the gallons of malaga, ſought). And ſo of the reſt.

e *For example:* A *tobacconiſt* would mix three ſorts of tobacco together: to-wit, one ſort of 18*d* a-pound; another, of 22*d*; and a third, of 2*s*. How much of each muſt he take, that the mixture may be ſold for 20*d* a-pound?—*Anſw.* (by linking the odd extreme to each of the others; and counterplacing the difference of the rates, twice to the odd-one:

$$d20\left\{\begin{array}{lr}18 & 2+4\\ 22 & -\,-\,2\\ 24 & -\,-\,2\end{array}\right\} i.e\left\{\begin{array}{c}6\\2\\2\end{array}\right\} \text{or}\left\{\begin{array}{l}\text{any numb.}\\ \text{in the ſame}\\ \text{proport. as}\end{array}\right.\left|\begin{array}{ccc}9 & 12 & 15\\ 3 & 4 & 5\\ 3 & 4 & 5\end{array}\right\}\&c.$$

For 6 : 2 : : 9 : 3, &c. in infinitum.

* The rule here is (indeed) limited, in its immediate effect, to the different anſwers found by the various methods of linking the ſimples; which can be done only a certain limited number of ways:—Yet, from this rule we can find an infinit number of other ſolutions. Thus, (1) Take any method of linking the ſimples: then take the quantities ariſing from that method; and, if you increaſe, or diminiſh each of them, in the ſame proportion (i.e. by equal multiplication, or diviſion) theſe new quantities are alſo true anſwers; for that very reaſon that they are proportional to thoſe ariſing immediatly from the linking, and differences: becauſe, if two quantities of two ſimples make a balance of gain or loſs, with reſpect to the mixture-price; ſo muſt double, or triple, or the half, or a third part, or any other proportion of theſe quantities. And, becauſe theſe quantities may be increas'd, or diminiſh'd, in an infinite variety of proportions; therefore it is plain, that we may proceed from an infinit variety of ſolutions. (2) Or if we only encreaſe, or diminiſh, the alternate, or correſpondent differences of any pair of ſimples, that are linkt-together; or of any two, or more pairs, leaving the reſt as they are: we may thus, alſo, proceed to an infinite number of ſolutions.

e *For example:* How much malaga, at 7*s* 6*d* the gallon; ſherry, at 5*s* the gallon; and white-wine, at 4*s* 3*d* the gallon, muſt be mixt with 18 gallons of canary, at 6*s* 9*d* the gallon; that the whole mixture may be ſold at 6*s* the gallon?—*Anſw.* (the differences being found, as afore, note [c]) As 12 (the quantity of canary, found by the differences of the rates) to 18 (the quantity of canary given) So (1) 21 (the quantity of malaga, found by the differences of the rates) to 31½ (the quantity of the malaga ſought) (2) 18 (as afore) to 27 gallons of white-wine, &c.

f *For example:* Suppoſe it were requird to mix malaga, at 7*s* 6*d* the gallon, with canary, at 6*s* 9*d* the gallon; ſherry, at 5*s* the gallon; and white-wine, at 4*s* 3*d* the gallon; ſo as that the whole mixture may be 90 gallons, to be ſold for 6*s* the gallon: How much of each ſort will compoſe the mixture?—*Anſw.*(the differences being found, as afore, n. [c]) As 60 (the ſum of the differences) to 90 (the quantity of the mixture) So (1) 21 (the difference, or quantity of malaga, found by the difference of the rates) to 31 ½ (the quantity of malaga for the mixture ſought) (2) 18 (the difference) to 27 (the quantity of the wine) &c.

Note—I. In mixtures, *one ingredient may be ſuch, as to bear no value in the mixture*; but only to encreaſe the quantity, and diminiſh the value. Therefore let its rate be repreſented by 0: as water mixt with wine; braſs, or other alloy, mixt with gold and ſilver. *For example:* (1) If 8 gallons of wine, at 9 ſhillings per gallon; 12 gallons, at 8*s*, are mixt together: how much water muſt be added to make the mixture worth only 6*s* per gallon? (*Anſw.*) I find the mixture-rate of the 8 gallons, and 12 gallons: then, I take 20 gallons at that rate, to mix with water, whoſe rate is 0; which is done according to the inſtructions in note [f]. (2) A goldſmith would mix gold 18 carats fine, 20 carats fine, 24 carats fine; and a quantity of alloy, to make the mixture 19 carats fine: How much muſt be taken of each? Repreſent the rate of the alloy by 0, and proceed as in caſe 2d.—II. Beſide the mixture of liquors, or any other kind of things, the ſame rules are applicable where *perſons* are the ſubjects. Thus: (1) 8 men, being boarded at the rate of 6*l* a quarter, for every man; 6 women, at 5*l* for each woman; and 4 children, at 2*l* for each child: How much does each perſon pay a quarter; taking them at an equal rate, one with another? This is plainly a queſtion of alligation medial; and to be ſolvd after that manner. (2) If the quarter's board for a man is 5*l*, for a woman 4*l*, for a child 3*l*, and for a ſervant 1*l*: How many men, women, children, and ſervants may be taken to board; ſo as their board, at an equal rate, may come to 3*l* 5*s*, for each perſon? This is alſo a plain queſtion of alligation alternate.—III. The rules of alligation may alſo be applied to the mixture of bodies, with reſpect to their *ſpecific gravities*, (or bulks, which is in effect the ſame thing) thus: (1) If there are given the ſpecific bulks of ſeveral bodies (or ſpecific gravities, by which the bulks may be found) together with the quantities or weights of each put into a mixture. (*Example*) Suppoſe 10 ounces of metal, whoſe ſpecific bulk is 3 (that is, 3 cubical inches to 1 ounce) are mixt with 14 ounces of another, whoſe ſpecific bulk is 2: What is the ſpecific bulk of the mixture? (*Anſw.*) $\frac{12}{29}$: found thus: If 1 ounce makes 3 cubical inches, 10 ounces will make 30: and, if 1 oz makes 2; then 14 oz make 28: and, laſtly, if 24 (*i. e.* 10 and 14) ounces make 58 (*i. e.* 30 and 28) cubical inches; 1 oz will make $\frac{12}{29}$. Then, the ſpecific gravities of the metals that are mixt, and of the compound, are $\frac{1}{10}$, $\frac{1}{14}$, $\frac{29}{12}$. (2) Having the ſpecific bulks (or gravities, by which the bulks are found) of ſeveral bodies to be mixt, and the ſpecific bulk (or gravity) to which a mixture of theſe bodies are to be reduc'd: To find the proportional quantities to be taken of each; that the mixture may bear the given rate of the ſpecific bulk (or gravity) Take the given ſpecific bulks of the bodies to be mixt: link them together: then take, and place their differences from the ſpecific bulk of the mixture, the ſame way as taught in alligation alternate; and you have the anſwer. (*Example*) Suppoſe two metals, whereof the one has 2 cubical inches to 1 lb weight, and the other 5: What proportion of weight of each muſt be taken, to make the mixture 3 cubical inches to 1 lb. (*Anſw.*) 2 lb of the 1ſt to every one of the other, as in the margin. (*Note*) By this rule is the famous queſtion ſolvd about Hiero, king of Syracuſe's crown: He gave a certain quantity of pure gold to make a crown: but ſuſpecting the goldſmith had mixt ſilver with it, he deſird Archimedes to diſcover it, if poſſible; who did it by this means: When he went into a bathing tub, he reflected that every body, immerſt in water, muſt put as much water out of its place as is equal, in bulk, to it-ſelf. Therefore he took a quantity of pure gold, and another of ſilver, each of the weight of the crown; or, as ſome ſay, cauſd a crown to be made of pure gold; and another, of ſilver; each of the ſame weight with the firſt crown; and meaſurd their ſpecific bulks by the bulks of the quantities of water put out of its place by the immerſion of each of the three crowns (or of the ſuſpected crown, and of each of the maſſes of pure gold and ſilver; which would put out the ſame quantity of water, whatever ſhape they were in) and, by compariſon of the three ſpecific bulks, he found how much

cub. inch.			lb weight
		2	2
	3		
		5	1

much gold and ſilver was in the mixt crown; which may be done after the manner of the example in note [e].

h The mixture of copper with gold and ſilver (calld the *alloy*) is partly to make thoſe metals harder for wear; and partly to defray the expenſes of coining; no ſeignorage * being now paid for coining mony.

* *Seignorage* is a duty belonging to a prince for the coining of mony; which has been different, in different nations, and reigns.——In England, in the 18th year of K. Charles II, it was aboliſht by an act of parliament; by which it was enacted that all mony ſhould be ſtruck at the public expenſe. So that weight is returnd for weight to all perſons, who carry their gold and ſilver to the mint.

i *That is,* If any quantity, or weight, of fine-gold be divided into 24 equal parts, (calld *carats*) and 22 of thoſe equal parts be mixt with 2 of the like parts of copper: that mixture is calld *Standard* gold: on which the goldſmiths generaly work.——*NB.* (1) The old ſterling, or right ſtandard gold of England (which obtaind till the 18th of Henry VIII) was 23 car. $3\frac{1}{2}$ grains of fine gold, and $\frac{1}{2}$ grain of alloy: which alloy, according to the red book, might be ſilver, or copper. (2) If, by aſſaying any quantity of gold by the coppel [or crucible, in which it is melted] it loſes nothing of its weight; it is fine gold *. If the loſs be $\frac{1}{24}$ part they call it 23 carats fine, or one carrat better than ſtandard. If it has loſt $\frac{2}{24}$ parts; it is 22 carats fine, or ſtandard: if $\frac{3}{24}$; it is ſaid to be 21 carats fine, or rather, one carat worſe than ſtandard: and ſo in proportion, as it happens to be better, or worſe.

* But it is obſervd, that, what care ſoever is taken in purifying *gold,* to clear it from droſs; it *can never be brought to* 24 *carats*; but, ſtill, comes ſhort of $\frac{1}{4}$ of a carat, or one grain. This grain they call a 16th: and this 16th they divide into two 8ths; and each of thoſe 8ths into two 16ths: on which calculation, they ſay gold may be purified as far as the firſt 16th of the ſecond 8th; but no further.

k *That is,* Eleven ounces, and two penny-weight of fine ſilver, and eighteen penny-weight of copper, being melted together, is eſteemd the true ſtandard for ſilver coin, calld *Sterling* ſilver. And ſo, in proportion, for a greater, or leſſer quantity, than for gold.

l The *value* is here given in round numbers: in the notes, more preciſely.——*NB.* In different reigns the value of gold and ſilver has been raiſd, and lowerd. V. Folkes, Leake, Nummi britannici hiſtoria, &c.

m A *pound* averdupois of copper is coind into 23 pence. Therefore (1) the Weight of a *half-penny* is 3.82 drams (2) and the Value of an *ounce* 5.75 farthings.

n By the indentures of the mint (43 El. 2 C. 1. 22 C. 2. 1 J. 2.) 62*s* were to be coind out of a *Pound*-troy of ſilver.——The true Weight, therefore (1) of a *crown* is about 1 dw. 8.496 gr. (2) of a *ſhilling* $92.903\frac{1}{5}$ grains.

o By the indenture of the mint (22 C. 2) $44\frac{1}{2}$ guineas were to be coind out of a *Pound*-troy of gold.——The true Weight of a *guinea*, therefore, is 129.43 gr.——and (at the rate of 21*s* per guinea) the Value (1) of an *ounce* is £ 3 18 1, 2.697 (2) of a *grain* q $7.812\frac{1}{2}$.

p Nearly. V. notes [c] [f] [g].——*NB.* *To find the Value of Foreign Coins*, according to the preſent value of the Engliſh (ſuppoſe, for inſtance, a piece 23 car. 2 gr. fine) Having found the fineneſs by the coppel——To know the *value of an ounce of it*, ſay: As 22 car. (the fineneſs of the Engliſh ſtandard) to 4 l (the value of an ounce of our coin) ſo is 23 car. 2 gr. (the fineneſs of the foreign coin). Then——For the *value of the coin*, ſay: If the value of 1 oz be £ 4 5 5 1.8; what will be the value of a coin, weighing ſo-much (as it is found to weigh).

ANNUITIES.

I. In Arrears.

I. *At Simple intereſt.*

1 A'NN [a] [in Arrears [b]] at Simple ínte-reſt: Fínd (I) the *Amount* [c], thus:

2 Súm of the ſéries to th' years, leſs 1 [d], into I [e] add N T. [f].

(II) *A'n-*

3 (II) *A'nnuity*: the Amount by Súm into Rate, more the years; gives [g].

4 (III) *Ráte*: A, leſs N′ into T, by N into Sum of ſer. leſs 1 [h].

5 (IV) *Time*: Take N′ into R′ (the firſt térm of a próg by that difference)

6 Thén take the Súm of the ſéries: Add N′ into númber more by one [i].

II. *At Compound intereſt.*

7 A'nn (in Arrears) at Compound íntereſt: Fínd (V) the *Amount*, thus:

8 Geo-prog Rate to the number o'th'years [k]: into N ſum o'th' ſeries [l].

9 (VI) *A'nnuity*: the Amóunt by th'Amount of 1′ (into th'Time) pound [m].

10 (VII) *Time*: A of P-correſpond by its P: Rate involve to the quotient [n].

Rate . . . . find by . . . . *Algebra* [o].

II. The PURCHASE,

1. For a CERTAIN NUMBER of YEARS.

I. *At Simple intereſt.*

11 Ann [Purcháſe] Simple íntereſt [p] (VIII) *Wórth*: Sum of diſcounts for each Year [q].

12 (IX) *A'nnuity*: given Worth by Wórth of 1 póund for years given [r].

13 (X) *Time*: N by ſeries of A of 1 póund; till the quóte gives Worth given [s].

II. *At Compound intereſt.*

14 A′nn (Purchaſe) Compound íntereſt (XI)
Wórth to find : príncipal-ſum find,
15 óf which the ′N's 1 year's íntereſt: Thén,
Worth of thát for the Time—Then,
16 Thís preſent Wórth from its príncipal—
Thĕ Remainder's the Worth of the N
ſought [t].
17 (XII) *A′nnuity′:* Worth of N ſought, b′y
Worth of 1 pound for Time [u].
18 (XIII) *Tíme:* Find a príncipal, whereóf 1
year's Intereſt is the N—Hence
19 Táke given Wórth—The remaínder's the
Wórth of that príncipal—Then, That
20 Príncipal by its Wórth gives the power of
the Tíme for the years ſought [w].

2. For Ever.

21 Ann (for Ever) at Cómp. int. (XIV) the
Worth, the Príncipal-ſum is
22 of which 1 year's íntereſt is the Rént, or N
given [x].
23 (XV) *Rate* (for ever) the Wórth more annuity
by the Worth, is [y].
24 (XVI) *Annuity* (for ever) the Wórth into Ráte
leſs the Worth, is [z].

3. In Reversion.

25 Ann (in Reverſion [aa]) at Compound intereſt:
To find (XVII) the *Value:*
26 (1) Firſt find the Worth (2) Then, what P
will amount to that worth, till Commencement [bb].

27 (XVIII) *Príncipal:* (1) Fínd A till cómm (2) Then what N will prodúce that A, for time [cc].

28 (XIX) *Time:* Find A till comménce. (2) Then, what T will gíve that A, for time [dd].

29 4. IN CASE OF FINES;

take the needful directions in Notes, as follow [ee].

a An *annuity*, or Penſion, is a ſum of money payable, every year, for a certain number of years, or for ever. And though it be divided into half year-, or quarterly payments *, it ſtill goes under the general name of annuity; becauſe the whole payments make ſo much in a year.

NB.—*The initial letters,* in the following queſtions and ſolutions, are the ſame as in the article Intereſt, note [a].—*For the readier calculating* of annuities, it is generaly thought the moſt convenient method, to have tables ready-made, extending to the greateſt number of years that ordinarily occur in that buſineſs; and for ſeveral rates of intereſt, that are moſt likely to occur: by means of which the anſwers of the moſt uſeful problems may be eaſily found *.—*But,* inaſmuch as the greateſt burthen of the work is the finding the power or product of the rate, or ſum of it, and 1 year's intereſt, multiplied continualy into itſelf, as the rules direct (which, if the number of years in the queſtion is great, becomes very tedious) it may ſuffice to give a table of the powers of ſeveral rates of intereſt carried to a convenient length; which are the amounts of 1 *l* principal for 1, 2, &c. years, at compound intereſt, according to problem V. †, That in the margin, at 5 per cent, to 31 years; is an example,

AMOUNTS of 1 *l* at 5 per cent, for a year, &c.	
1	1.05
2	1.025
3	1.157525
4	1.215506
5	1.276281
6	1.340096
7	1.407100
8	1.477455
9	1.551328
10	1.628895
11	1.710339
12	1.795856
13	1.885649
14	1.979932
15	2.078928
16	2.182874
17	2.292018
18	2.406919
19	2.526950
20	2.653298
21	2.785962
22	2.925261
23	3.071524
24	3.225100
25	3.386355
26	3.555673
27	3.733456
28	3.920129
29	4.116135
30	4.321942
31	4.538039

which may be extended to more years; and the like made for other rates of intereſt ∔. V. Fractions, note [n].

* The problems, that follow under this head, are calculated on the ſuppoſition of annual payments: but the rules are applicable to half-yearly or quarterly-paid annuities, the ſame way as to yearly payments: only taking T to repreſent the number of half-years, or quarters, that an annuity continues; R, the intereſt of 1 *l* for $\frac{1}{2}$ or $\frac{1}{4}$ of a year; and N, the half-year's, or quarter's payment.

† The tables, which are common upon this ſubject, are limited to 1 *l*.—Thus, we have tables (1) of the *amounts* of 1 *l*, for 30 or 50 years, at ſeveral rates, compound intereſt. (2) of the preſent *worths* of 1 *l*, due after any number of years, from 1 to 30 or 50. (3) of the amounts of 1 *l annuity*. (4) of the preſent worths of 1 *l* annuity (5) of the annuity to be purchas'd for 1 *l*.—By means of which the anſwer to the ſeveral problems is not a little facilitated.

∔ *NB*. This table was made by completing the multiplication at every ſtep; and then taking the firſt 6 decimal places. Alſo, when the figure in the 7th place exceeded 5, 1 was added to the 6th place: which makes the error leſs; only it makes it exceſſive, inſtead of defective.

b An annuity is ſaid to be *in arrears*; when the debtor keeps it in his hand for a certain number of years, paying the whole at laſt with intereſt, for every year after it falls due.

c The amount of an annuity, forborn for any number of years, is the total of the ſeveral years with the intereſt due upon each—*NB*. If an annuity is to be bought off, or paid all-at-once, at the very begining of the firſt year; the price, which ought to be paid for it, diſcounting for the advancement, is calld the *preſent worth* of it for ſo many years.

d *For example*. Seven years: the ſum of the ſeries, leſs 1 *, is $1 + 2 + 3 + 4 + 5 + 6 = 21$: found (more readily) by progreſſion, thus: $1 + 6 = 7$: then 6 (the ſeries) × 7 the ſum of the extremes 1 and 6) $= 42 \div 2 = 21$.

* Whatever the time is, there is due upon the firſt year's annuity as many years intereſt as the whole number of years leſs; and gradualy 1 leſs upon every ſucceeding year, to the laſt but one: upon which there is due one

year's intereſt, and none upon the laſt. Wherefore, in the whole, there is due as many years intereſt of the annuity, as the ſum of the ſeries, 1, 2, 3, &c. to the number of years leſs one.—Conſequently, one year's intereſt, multiplied by this ſum muſt be the whole intereſt due: to which the whole annuities being added; the ſum is plainly the amount.

e The *intereſt* of the annuity is—the rate; if the annuity be 1 *l*—the product of the rate and annuity; if the annuity be more than 1 *l*.

f *That is*, the product of the annuity and time, or the whole annuity.

For example. [Given n t r; to find A] What is the amount of 50 *l* annuity for 7 years, allowing ſimple intereſt, at 5 *l* per cent, for every year after it falls due? —*Anſw.* $[1s^{t} - 1 \times i + nt = a]$ 21 (the ſum of the ſeries of the number of years, leſs 1) × 2.50 (the product of the rate and annuity) = 52.5 (the whole intereſt due upon the annuity) + 350 (the product of time and annuity) = 402.5: that is, 402 *l*, 10 *s*; the amount ſought.

g *For example:* [Given a r t; to find N] What annuity will, in 7 years, amount to 402 *l* 10 *s*: allowing 5 per cent, ſimple intereſt?—*Anſw.* $\left[\frac{a}{sr+t} = n\right]$ 402.5 (the amount) ÷ 8.05 (the ſum of the ſeries 21) × .05 (the rate) + 7 (the time) = 50 *l*; the annuity ſought.

h *For example:* [Given a n t; to find R] At what rate of intereſt will an annuity of 50 *l* amount to 402 *l* 10 *s*, in 7 years, at ſimple intereſt?——*Anſw.* $\left[\frac{a-nt}{ns} = r\right]$ 402.5 (the amount) — 350 *l* (the annuity 50, × 7, the years) = 52.5 ÷ 1050 (the annuity 50 *l*, × 21 the ſum of the ſeries, leſs 1 †) = .05, the rate ſought.

† Viz. 1 + 2 + 3 + 4 + 5 + 6 = 21.

i *For example:* [Given a n r; to find T] In what time will 50 *l* annuity amount to 402 *l* 10 *s*, at 5 per cent.

Anſwer. 50 × .05 = 2.5 for the firſt term of a progreſſion.

	rn	2 rn	3 rn	4 rn	5 rn	6 rn
	2.5 +	5 +	7.5 +	10 +	12.5 +	15
Sums :		7.5	15	25	37.5	52.5
× 50 :		150	200	250	300	350
Sums :		157.5	215	275	377.5	402.5

Then, proceed

ceed as in the margin: where it may be obſervd (1) that 2.5 is the common difference of the progreſſion: for, 2.5 × 2.5 = 5: and 5 + 25 = 7.5: &c. (2) that 2.5 (the 1ſt term) × 5 (the 2d term) = 7.5 (plac'd under the 2d term) and 25 + 5 + 75 (advancing to the third ſtep) = 15 (plac'd under the third term) &c. (3) that 50 (the annuity) × 3 (the number of the terms ſumd, more by 1) = 150 (plac'd under the ſecond term) and, likewiſe, 50 × 4 = 200 (under the third place) &c. (4) that the ſums of the ſeries, and the products of 50, added together, bring us (in courſe, at the fifth ſtep) to a ſum equal to the given amount. (5) that the number multiplied into the annuity, in that laſt ſtep (viz. 7) is the time ſought. —*NB*. If you never find a ſum equal to the amount; then the problem is impoſſible in whole years.

k *That is*, of the geometrical progreſſion—Make 1, the leaſt term; the rate, the ſecond; which conſequently, is the ratio of the progreſſion, by which every term is to be multiplied, to produce the next.—— Carry this to as many terms as there are years——Its ſum is the amount of 1 *l* annuity for the given time.

l The ſum of the ſeries for the given time *, multiplied into the given annuity, gives the amount ſought.

For example: [Given n r t; to find A] What is the amount of an annuity of 40*l*, to continue 5 years, allowing compound intereſt at 5 per cent. *Anſw*. $\left[\frac{rt}{r-1} \times n = a\right]$ Of a geometrical progreſſion, begining with 1, whoſe ratio is 1.05, the fifth term is $1.\overline{05}{}^{4}_{1} = 1.21550625$: and the ſum of the ſeries is 5.52563125 †. which multiplied into 40, the product is 221.02525, the amount of 40*l* annuity.

* *To find the ſum* moſt eaſily; multiply the laſt term into the rate (or ratio) which produces a power of the rate, whoſe index is the time; and, from the product take 1, the firſt term: then, divide the remainder by the rate leſs 1: the quote is the ſum. V. note †.

† For, $\overline{1.05}^{5} = 1.2762815626$; and $\overline{1.05}^{5} - 1 = 2762815625$: which divided by 1.05 —1, or .05; the quote is 5.52563125, the amount of 1 *l* annuity for 5 years.

m *For example*: [Given a r t; to find N] What annuity will amount to 221*l* 6*d*, in 5 years, at the rate of 5 per cent, compound intereſt? *Anſw.* $\left[\frac{a}{a\ 1}\right]$ 221.025 (the amount given) ÷ 5.52563125 (the amount of 1 *l* annuity in 5 years) = 39·999 = 39*l* 19*s* 11*d* 3*q*, &c. or 40*l* nearly: which it would have been preciſely; had we taken 221 *l* 0*s* 6.06 *d* (or 221.02525) for the amount, as in the preceding problem.

n *For example:* [Given a n r; to find T] In what time will 40*l* annuity amount to 221.02525*l* at the rate of 5 per cent—*Anſw.* To find a correſponding principal, ſay .05 : 1 : : 40 : 800. Then 800 (the correſponding principal) + 221.02525 = 1021.02525; which, divided by 800, the quote is 12762815625; equal to the fifth power of 1.05, or $\overline{1.05}^{5}$: ſo that 5 is the number of years ſought.

o Here, and in the following problems, I might ſay, with Mr Malcolm (p. 608) that, There is no rule, within my limits, that will ſolve this problem—But, as Mr Hill (p. 343) has helpt us to an arithmetical expedient; the reader will not be diſpleasd with a clearer and conciſer account of it; which I have given under the head of Approximation.

p The rule, here given, is that of Kerſey, ſupported againſt Moreland, Ward, &c. by Malcolm: and is certainly true, in conſiſtence with the ſuppos'd condition, or agreement of the allowing ſimple intereſt; but not abſolutely ſo. For, if we enquire what (in ſtrict equity, and juſtice) ought to be paid for the annuity; then, the rule does not ſhow it: ſince it gives too-much. The true price, Mr Malcolm (arithm. 6.10. p. 600) obſerves, muſt be found by diſcounting compound intereſt. Nor is this contrary to law: for, though, when an annuity is in arrears, the law forbids taking compound intereſt; yet, in the purchaſe of an annuity, if the buyer offers ſuch a price, as allows him compound intereſt for the advance of his mony; he does nothing contrary to the law; becauſe, in buying, a man may offer what price he thinks fit: and he has this good reaſon for it, that, by putting-out his mony, and lifting it, at every year's end, he can improve it by compound intereſt. But, to ſhow, further, how unjuſt ſimple intereſt is, in the purchaſe of annuities; Mr Malcolm gives us an example An annuity of 50*l* is to be bought for 40 years, diſcounting ſimple intereſt, at 5 per cent. The price, according to Moreland's rule, is 1316*l* 13*s* 4*d*: a ſum, of which one year's intereſt exceeds the annuity. Would not one think, then, he had made a pretty bargain, to give, for an annuity to continue only 40 years, a ſum, which would yield him a greater yearly intereſt for ever? If it is alſo calculated by the other rule; the ſame will happen, as (he tells us) he has actualy found: though it is much leſs than the other; as it does not exceed 1100.

q *For example:* [Given a r t; to find W] What is the preſent worth of an annuity of 100*l*, to continue 5 years, diſcounting at the rate of 6 per cent, or .06 to 1 *l*—— *Anſw.* Find the preſent worth of each year, by itſelf, diſcounting from the time it falls due (V. Rebate) the ſum of all theſe is the preſent worth ſought.

	years		as	to	ſo is	to	
Thus the Amount of 1 *l*... for	1	is	1.06	1	100	94.33962	&c.
	2		1.12	1	100	89.28571	
	3		1.18	1	100	84.74576	
	4		1.24	1	100	80.64516	
	5		1.30	1	100	76.92307	
						425.93932	

NB. The work will be ſomewhat eaſier, if you find the preſent worth of 1 *l* annuity for the given time; and, then, multiply that by the given annuity. The product is the preſent worth: becauſe of the proportionality of annuities, and their preſent worths.

r *For example:* [Given r t w; to find N] What annuity, to continue 5 years, is worth 220*l* preſent worth; allowing ſimple intereſt at 5 per cent?—*Anſw.* 4.3641 (the preſent worth of 1 *l* for 5 years) ÷ 220 (the preſent worth of the annuity ſought) = 50.399, the annuity ſought.

s *For example:* [Given n w r; to find T] What time muſt an annuity of 50*l* 8*s* continue, to be worth 220*l* ready mony, at the rate of 5 per cent?—*Anſw.* 50.4 (the annuity) ÷ 1.05 (the amount of 1 *l* for 1 year) = 48. And 50.4 ÷ 1.10 (the annuity of 1 *l* for 2 years) = 45.818. And 50.4 ÷ 1.15 (the amount of 1 *l* for 3 years) = 43.827. And 50.4 ÷ 1.20 (the amount of 1 *l* for 4 years) = 42. And 50.4 ÷ 1.25

(the amount of 1 *l* for 5 years) = 40.32 —— Then 48 + 45.818 + 43.827 + 42 + 40.32 = 219.965. —— This ſum, then, being nearly the value of 220, the preſent worth given: the number of quotes (to-wit, 5) is the number of years ſought.

t *For example:* What is the preſent worth of an annuity of 40*l*, to continue 5 years, diſcounting at 5 per cent? ——*Anſw.* (1) To find a principal ſum, whereof 1 year's intereſt is the annuity (to-wit, 40*l*.) Say, .05 : 1 : : 40 : 800. Then (2) To find the preſent worth of 800*l* (the principal ſum found) Say 1.05 (the amount of 1 *l* for 5 years) involv'd 5 times, = 1.276281 (the preſent worth of 1 *l* for 5 years) and 800 ÷ 1.276281 = 626.8212 (the preſent worth of 800*l*) (3) Then 626.8212, ſubtracted from 800 (its principal) gives 173.1788 (the preſent worth of the annuity ſought).

u *For example:* What annuity, to continue 5 years, will be purchasd for 173*l* 3*s* 7*d*; allowing compound intereſt at 5 per cent?—*Anſw.* 173.1788 (the preſent worth of the annuity ſought) ÷ 4.329 (the preſent worth of 1 *l* annuity, 5 years) = 40.004 (the annuity ſought).

w *For example:* What time muſt an annuity of 40*l* continue, to be worth (of ready mony) 173*l* 3*s* 7*d*; allowing 5 per cent, compound intereſt?—*Anſw.* (1) 800*l*. (the principal ſought *)—173.1788 (the given worth) = 626.8212 (the preſent worth of 800*l*, due at the end of the time ſought) Then 800 ÷ 626.8212 = 1.27678 (the power of the 5th root, nearly) So, the time ſought is 5 years.

* Found, as in problem XI.

x *For example:* What is the price of a perpetual annuity of 40*l*, diſcounting at 5 per cent, compound intereſt? —-*Anſw.* $\left[\frac{n}{r-1} = \underset{\bullet}{W}\right]$ 1.05 (the rate)— 1 = .05. Then, As .05 (the rate leſs 1) to 1: So is 40*l* (the annuity) to 800*l* (the price, or preſent worth of the annuity to continue for ever)—*Note*, The reaſon of this rule ſeems of its ſelf obvious. For, it is plain, that, Since a year's intereſt of the price is the annuity: therefore there can neither more, nor leſs, be made of the price than of the annuity; whether we employ it by ſimple, or compound intereſt.

y *For example:* In the preceding queſtion [given w n;

to find R) Say $\left[\frac{w+n}{w}=r\right]$ 800 (the worth) + 40 (the annuity) = 840 ÷ 800 = 1.05 (the Rate.)

z *For example:* In the preceding queſtion [given w r; to find N) Say [wr—w = n] 800 (the worth) × 1.05 (the rate) = 840 —800 (the worth) = 40 *l* (the annuity).

aa An annuity or yearly rent, *in reverſion,* is, when it is not to be enterd upon till after ſome time, or number of years are paſt.

bb *For example:* Suppoſe it were requird to compute the preſent worth of 75 *l*, yearly rent; which is not to commence, or be enterd-upon, till ten years hence, and then to continue ſeven years after that time; at 6 per cent, and compound intereſt?—*Anſw.* (1) Firſt (by problem XI) the preſent worth of 75 *l*, to continue 7 years at 6 per cent compound intereſt, will be found to be 418.6783. (2) Then (by problem XII) the principal of that worth (viz. 418.6752 *l*) or the ſum it will amount-to, put-out 10 years, will be found to be 233.7877. That ſum, therefore, is the preſent worth of 75 *l* per annum, in reverſion, &c. as was requir'd.

cc *For example:* What annuity, or yearly rent, to be enterd-upon ten years hence, and then to continue ſeven years; may be purchasd for 233 *l* 15 *s* 9 *d*, ready mony; at 6 per cent, and compound intereſt?—*Anſw.* (1) Firſt (by V) the amount of 233.7877 *l* (the preſent worth of an annuity to be enterd-upon after 10 years, at 6 per cent, compound intereſt) will be found 418.6752 *l*. (2) Then (by VI) the yearly rent (which, being ſold, will produce that amount for the preſent worth, at the ſame rate, and forborn 7 years) will be 75 *l* (the annuity ſought).

dd *For example:* The preſent worth of a leaſe in reverſion is 233.7877 *l*. The leaſe is 75 *l* a-year; and commences 10 years hence; and the allowance to the purchaſer is 6 per cent. I would know the time of its continuance.—*Anſw.* (1) Firſt (by V) the amount of 233.7877 (the worth given) is 418.6752. (2) Then (by VII) the time (which the yearly rent, given, is ſold-for, to produce that amount for the preſent worth at the ſame rate) will be 7 years.

ee Annuities, and rents of houſes, or lands, are of the ſame nature; wherein the ſame queſtions occur as to their being in arrears, or being purchasd—But, with reſpect to leaſes, there ariſe queſtions with ſome different circumſtances, owing to the practiſe of taking what they call *fines*: which is a ſum of mony paid at the begining of the leaſe, beſides the yearly rent —The following queſtions, with a general direction for the ſolution, will be ſufficient on this ſubject.

I. There is a piece of land worth 20 pound yearly rent, and 100*l* fine, for a leaſe of 21 years. The maſter is willing to quit the fine, and increaſe the rent: What ought the rent to be?—*Rule*. Find what rent, or annuity, to continue 21 years, 100*l* will purchaſe, diſcounting at the agreed rate of intereſt. The ſum of that, and the former rent, is the rent ſought.——*Obſerve*. If the whole is not to be taken-away; find the annuity anſwering to the part taken-away.

II. A piece of land is worth 12*l*, yearly rent, and a fine of 30*l*, for 19 years. The farmer is willing to pay more fine; and reduce the rent to 20*l*. What ought the fine to be?—*Rule*. Take the difference of the two rents (30 and 20) and find the preſent worth of an annuity equal to the difference for the ſame time (19 years.) That is the additional fine to be paid.—*Obſerve*. The ſame way the whole rent may be taken-away.

III. There is a farm to be let for 21 years, at 10*l*, yearly rent; and 20*l* fine. If the ſame be let for 30 years at the ſame rent; What ought the fine to be?—*Rule*. Find what annuity 20*l* will purchaſe for 21 years, at the agreed rate. That, added to the rent 10*l*, is the true rent with no fine. Therefore, find the preſent worth of that annuity, to continue 30 years: it is the fine ſought.

IV. A perſon has ſeven years to run of a leaſe of 21 years; for which he paid 40*l* fine, and 15*l* yearly rent. He would renew the leaſe to 16 years from this time; that is, for 12 years after the firſt leaſe expires. What fine ought he to pay?——*Rule*. Find what rent, for 21 years, the given ſum 40*l* will purchaſe: Then, find the preſent worth of this yearly rent to continue 12 years. Laſtly, find the preſent worth of this laſt preſent worth; rebating for 7 year's time, that remains of the old leaſe. This is the fine to be paid.

APPROXIMATION.

1 APPROX. a branch of Falſe, for the finding the *Rate* of annuities [a].

a *Rate to find*, in the doctrin of annuities, is a problem beſt ſolvd by algebra (V. Annuities, note [o]) But, for the ſake of ſuch as are unacquainted with that ſcience, I have here added, what Mr Hill (p. 343) recommends, in this caſe, the expedient of *approximation*; which he looks-upon as the moſt conciſe, and quickeſt method we can uſe.—The procedure is as follows:

In caſe of Arrears (annuities, amount, and time, being given) make two or three trials, till you get the anſwer betwixt two of the neareſt reſults. The nearer the reſults approach to the amount, the better. And, intereſt being ſeldom above 10 *l* or under 5 *l* per cent, a ſuppoſition between theſe two cannot err much from the truth. Then the work may be performd by proportion——*For example:* An annuity of 20 *l* per annum is offer'd to be let for 180 *l* 10 *s*, to be paid at the end of the ſaid term: What intereſt is allow'd in this bargain?—*Anſw.* Firſt I make a ſuppoſition at 8 per cent; and, by problem V, (Annuit.) I find the amount, at that rate, to be 178.456; which is too little by 2.044 *l*. . . Wherefore, becauſe I ſee I am near, I make my ſecond trial at 8 *l* 10 *s* per cent: and, working as before, I find the amount to be 181.21; whereby I ſee I have overſhot the truth by 71; and I ſee the anſwer is bounded betwixt 8 *l* per cent, and 8 *l* 10 *s* per cent. . . By theſe two ſuppoſitions, and their reſpective errors (as in the rule of Falſe) I find the rate as follows: (1) As 2.745 (the ſum of the errors; to wit, 2.044 + 71) to 5 (the difference of the ſuppoſitions; to-wit, 8 + 8.5) So is 2.044 (the former error) to .371: which, becauſe leſs than the true amount, being added to the former ſuppoſition, gives 8.371 (the rate of intereſt ſought: (2) Or, As 2.745 to .05: So is 71 (the latter error) to .129: which, becauſe more than the true amount, being ſubtracted from the latter ſuppoſition gives 8.371; the rate of the intereſt ſought, as afore.

In caſe of Purchaſing (annuity, preſent worth, and time of continuation, being given) proceed as before——*For example:* An annuity of 20 *l* per annum, to continue for 7 years, is ſold for 100 *l* ready mony. What rate of compound intereſt has the purchaſer, for the mony ?—*Anſw.* (1) Firſt I make my ſuppoſition at 9 per cent : and (by problem XI) I find the preſent worth of 20 *l* per annum, to continue 7 years, is 100.659056 ; which ſhould have been 100 *l* : wherefore the error is .659056. (2) Then, ſeeing the ſuppoſition was ſhort, I make my next ſuppoſition at 9 *l* 5 *s* : and (by the ſame problem) I find that the ſaid annuity, for the ſame time, will be worth 99.82 *l* ; which ſhould have been 100 *l* : by which I ſee that I have ſuppoſd too-much ; and the error is .18. (3) Then, As .839 (the ſum of the errors) to .25 (the difference of the ſuppoſitions) So .18 (the latter error) to .0536 : which ſubtract from the former ſuppoſition (becauſe it was too-great) and there will remain 9.1964 (or 9 *l* 3 *s* 11 *d*) for the rate ſought.

BARTER.

1 BA′RTER, exchánge of commodities : the
rúle to propórtion 'em ăs follows :
2 Whát's to be chángd, Value : thén, ſee what
That will púrchaſe of T'other [a].
3 I'f an advánc'd price of óne, a propórtionable
fínd for the other [b].

a *For example:* Two merchants barter. *A* would exchange cw 5 3 14 of pepper (worth 3 *l* 10 *s* a-pound) with *B*, for cotton (worth 10 *d* a-pound). How much cotton muſt *B* give to *A* for his pepper ?——*Anſw.* (1) Find the value of the pepper, thus : As 1 lb to £ 3 10 : : cw 5 3 14 to £ 20 11 3 (2) Then ſee how much cotton that will purchaſe, thus : As 10 to 1 lb : : £ 20 11 3 to cw 4 1 17½. (3) And ſo much muſt *A* have for his cw 5 3 14 of pepper.

b *For example:* Two merchants barter, thus : *A* has 86 yards of cloth, worth 9 *s* 2 *d* a-yard, ready mony ;

but, in barter, will have 11 *s* a-yard. *B* has ſhaloon, worth 2 *s* 1 *d* a-yard, ready mony. Now, how-many yards of the ſhaloon muſt *B* give, to make his gain, in the barter, equal to that of *A*'s?——*Anſw.* (I) Having firſt found what advance *B* ought to make per yard on his ſhaloon, in proportion to what *A* has done upon a yard of his cloth, thus: As 9 *s* 2 *d* (the worth of the cloth per cloth per yard) to 11 *s* (the advanc'd price) ſo 2 *s* 1 *d* (the worth of the ſhaloon per yard) to 2 *s* 6 *d* (the advanc'd price of the ſhaloon.) (II) Say, as before, note [a]. (1) As 1 yard to 11 *s* (the advanc'd price of one yard) ſo is 86 yards (the whole of the cloth) to 946 *s* (the advanc'd value of all the cloth). (2) Then, If 2 *s* 6 *d* will buy 1 yard of ſhaloon (at its advanc'd price) how-many yards will 946 *s* buy? the anſwer will be $378\frac{2}{5}$ yards: which is what *B* is to give of his ſhaloon, to *A*, for his 86 yards of broad cloth.

COMBINATION.

1 COMB. or how-óft any number of things
may chánge their poſition [a].
2 *Changes*: — ónce 2 is 2.—then, 2 into 3 is
6 — then
3 6′ into 4's 24′— and ſo ón [b], to the térm
to-be-varied [c]:
4 So L--éf is ádz [d]: P--oidy, uzoz: N--oztez,
taue: eieiyz [e].
5 If two, or more, are the ſáme; the *whole*
chánges by thóſe of the *ſame* Quote [f].
6 *Choices*. (1) OF, to the number *up* multiply.
(2) IN, to the ſame number, *downward*.
(3) This by That divide: the quotient
gives the choices [g].
7 *Compoſitions*: Of 2 ín 10, Square; óf 3 in
10, Cube the numbers [h].

Combinations are diſtinguiſht into changes, choices, and compoſitions—*Changes* (Permutations, Variations, Alternations) are ſuch combinations of any number of things, wherein reſpect is had to the order of the whole, either as to place, or ſucceſſion. (1) In regard of *place*; when any number of things being propoſd, the number of different ways theſe things may be diſpos'd in an equal number of determind places, ſo that they ſhall never be all in the ſame places. As, Suppoſe 6 things, A, B, C, D, E, F, are to be diſpos'd in 6 places: This may be done various ways, according to the different places every one may poſſeſs, regard being ſtill had to the whole: *i. e.* if any two, or more of them change places: that makes a new alternation, or order of the whole; though all the reſt remain unchang'd. (2) In regard of *ſucceſſion*; when ſeveral things may be taken, or orderd in ſucceſſion one after another. For example, In the taking of A, or B, or any one of them, 1ſt, or 2d, &c. And, as the taking any one of them 1ſt, or 2d, &c. may be calld puting them in the 1ſt, or 2d, &c. place of the ſucceſſion; this ſhows the coincidence of theſe two ways of ordering things as to the number of changes: for they are reducible to one notion of place, either as it relates to ſpace (which is more ſtrictly calld place) or to time, or ſucceſſion; which, as to the number of changes, is the ſame. For, places cannot be better diſtinguiſht than by numbering them 1ſt, 2d, 3d, &c. and the order of ſucceſſion of things is diſtinguiſhable no other way than by marking which thing is 1ſt, 2d, 3d, &c.——*Choices* (or Elections) are combinations, which regard not the order of the whole; but the way of taking a particular number out of the whole. Thus, ſuppoſe a leſſer number of things is to be taken out of a greater, and we are at liberty to take them out of any part of the whole; the number of ways this may be done, ſo that ſome (one, at leaſt) ſhall be different in every choice or combination, is calld the choices of that number of things in the other. For example, If 4 men are to be drawn out of a 100, the number of ways this can be done, ſo as ſome one of them ſhall be a different man, is the choices of 4 men (or any other thing) in 100.——*Compoſitions* are limited elections. Thus, conceive two or more different ſets (or ſyſtems) of things, containing each the ſame, or a different number of things: then, ſuppoſe we are to chooſe, out of the whole, a number of things either equal, or unequal to the number of ſets, ſo that we take ſome part out of every ſet *; the number of choices, thus limited, is calld the compoſition of that number of things, out of that number of ſets. For example: Suppoſe 16 companies of men; 16 men may be drawn out of theſe, various ways; taking only 1 man out of each company: and the number of choices we can make (with this limitation, of one out of each company) is the compoſitions of 16 in the 16 companies.

* If poſſible: that is, if the number of ſets be greater than the number to be choſen. For, then, we may take any choice of a number of the ſets, equal to the number to be elected.

b *That is,* Reſuming the laſt product (24) and multiplying it into the next digit (5) &c.

c *For example:* In this verſe ('Tu in me ita es, hem! in te ut ego ſum: ac tu me ibi ama, ut te ego amo hic jam') the words being 22, may be plac'd 1,124.000, 727.777,607.680,000 different ways.

d *That is,* 5 things multiplied into 24 (the product of the preceding multiplications, or the changes of 4) admit of (adz) 120 variations: and, conſequently, 6 (6 × 120) makes (oidy) 720: and ſo of the reſt.

e And thus may be known (without begining the ſeries) any changes, as far as 10. For (N--oztez, taue . . ei eiyz) 362,880 (the changes of 8) × 9 = 3.628,800; and ſo on *.

Examples: (I) Hereby may be known how many ways the letters of a name, or word, may be varied; and differently diſpoſd by way of anagram: out of which thoſe of uſe may be gatherd, neglecting the reſt. As, for example: the word 'Roma', conſiſting of 4 different letters, may admit of 24 changes, as in the margin: of which only ramo, oram, mora, maro, armo, amor, are ſignificant †. (II) Six gentlemen, that were traveling, met-together (by chance) at a certain inn upon the road; where they were ſo pleasd

Roma	Orma	Mroa	Arom
Roam	Oram	Mrao	Armo
Rmoa	Omra	Mora	Aorm
Rmao	Omar	Moar	Aomr
Raom	Oarm	Maro	Amro
Ramo	Oamr	Maor	Amor

pleasd with their hoſt, and each other's company; that, in a frolic, they made a contract to ſtay at that place, ſo long as they, together with their hoſt, could ſit every day in a different order, or poſition, at dinner. This will be found near 14 years. For, they, being made 7 with their hoſt, will admit of 5040 different poſitions: but 5040, being divided by $365\frac{1}{4}$, the number of days in one year, will give 13 years and 291 days. (III) In St. Mary le Bow's church, Cheapſide (before the fire, in 1666) were 12 bells. Suppoſe it were requird to tell how many ſeveral changes might have been rung upon thoſe 12 bells; and, at a moderate computation, how long all thoſe changes would have been ringing once over. In Anſwer, (1) Firſt $1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 \times 10 \times 11 \times 12 = 479001600$ the number of changes. (2) Then, ſuppoſing there might be rung 10 changes in one minute (to-wit, $12 \times 10 = 120$ ſtrokes in a minute, which is two ſtrokes in a ſecond of time) according to that rate, there muſt be allowd 47900160 minutes, to ring them once over ‡. (IV) The two following verſes [Lex, rex, grex, res, ſpes, jus, thus, ſal, ſol, (bona) lux, laus. Mars, mors, ſors, fraus, fæx, ſtyx, nox, crux, pus (mala) vis, lis] the ſenſe remaining good, and the verſe true, may be varied 79833600 ways §: which would compoſe above 249 volumes; each volume containing 2000 pages; every page divided into two columns, and each column to contain 80 verſes; which, at a penny the ſheet, would amount to 518 pounds, 15 ſhillings; and, ſuppoſing them bound for 5 ſhillings a volume, the binding would coſt 62 pounds, 5 ſhillings; and the worth of the whole would be 581 pounds. (V) The 24 letters of the alphabet will admit of 620448401733239439360000 ſeveral variations. If, with theſe, you take-in the ſeveral variations of 2, 3, 4, 5, 6, &c. letters; there will ariſe thence ſuch a vaſt number of words, that if a man could read 50 thouſand words in an hour, which is more than the pſalms of David contain; and, if there were a hundred thouſand millions of men, they would not ſpeak theſe words, according to their hourly proportion beforementiond, in a hundred thouſand years; a thing ſeeming moſt impoſſible and incredible, yet moſt certain and infallible in computation.

* Thus we learn how to find the number of changes of any number of things. But, if it ſhould alſo be requir'd actualy *to take them all out*, or repreſent them (for example) by letters: there is one certain method of proceeding, by which we can go through the whole, with the greateſt eaſe and diſtinctneſs; ſo as to run no hazard (or the leaſt poſſibility) of omiting any change, or taking any one oftener than once. This method will be made clear by a few examples: (I) For two things, a b; the changes are 2; a b, and b a.

(II) For three things, a b c; the changes are 6, which you ſee already taken-out in the margin; only, to ſave ſuperfluous writing, they may be orderd as above: where becauſe every letter poſſeſſes the firſt place twice (to-wit, till the remaining two have chang'd twice) therefore I write that letter down but once in the 1ſt place, ſuppoſing it to belong to the 1ſt place of the next change, which is left not filld-up. (III) For 4 things, a b c d; the changes are 24, as they are here repreſented: where every letter poſſeſſes the firſt place 6 times; (to-wit) till the remaining 3 have chang'd 6 times: whoſe changes are orderd the ſame way as in the preceding example.

a	b	c	b	a	c	c	a	b
	c	b		c	a		b	a

a	b	c	d	b	a	c	d	c	a	b	d	d	a	b	c
		d	c			d	c			d	b			c	b
	c	b	d		c	a	d		b	a	d		b	a	c
		d	b			d	a			d	a			c	a
	d	b	c		d	a	c		d	a	b		c	a	b
		c	b			c	a			b	a			b	a

(IV) For 5 things; a b c d e, their changes are 120 (= 24 × 5) taken as here repreſented. Where obſerve, that, becauſe the changes of 4 are 24; ſo, in taking-out thoſe of 5, every letter muſt poſſeſs the 1ſt place 24 times;

A	b	c	d	e	c	b	d	e	d	b	c	e	e	b	c	d
			e	d			e	d			e	c			d	c
		d	c	e		d	b	e		c	b	e		c	b	d
			e	c			e	b			e	c			d	b
		e	c	d		e	b	d		e	b	c		d	b	c
			d	c			d	b			c	b			c	b

B	a c d e	c a d e	d a c e	a c d
	e d	e d	e c	d c
	d c e	d a e	c a e	c a d
	e c	e a	e a	d a
	e c d	e a d	e a c	d a c
	d c	d a	c a	c a

And ſo on,

i. e. till the remaining 4 letters make 24 changes: which are taken-out according to the method of example 3. ——*NB.* (1) By theſe examples, *the method for any other number* may be eaſily underſtood; one depending always upon the preceding. So, if there were 6 things, whoſe changes are 720 (= 120 x 6) each of them muſt poſſeſs the 1ſt place 120 times; to-wit, till the remaining 5 make their 120 changes. And, obſerve, that, as every letter has the 1ſt place as oft as the changes of the remaining number; ſo, while it poſſeſſes the 1ſt place, the letter next it (in the firſt change of thoſe wherein it has the 1ſt place) poſſeſſes that next (or 2d) place, as oft as the number of changes of the remaining letters after this one: and, then, the next letter is advanc'd into that 2d place: and ſo on; till they are all ſucceſſively in the 2d places. The ſame is to be obſerv'd of the 3d, and 4th, &c. places. Then, when all the letters, after the 1ſt, have poſſeſt the 2d place; a new letter is-advanc'd into the 1ſt place: and, ſo, the changes proceed with that letter in the 1ſt place, as it did before. (2) But, it may be obſervd, that, *The number of changes grow ſo faſt* upon the ſeries of numbers, that the changes of a ſmall number of things can never be all repreſented. For example: The changes of 10 are 3628800: and allowing a man to take-out 300 of them every hour, it would coſt him 304 days to finiſh them all; though he workt-at-it night and day, without interruption. But, if we only double the number of things; *i. e.* take 20: the changes are 187,146.308,321.280,000; ſo great a number, that, if a man could take-out 500 of them every hour, it would take him upwards of 42 thouſand million of years to finiſh them all. For, divide the changes by 500, the quote is 374.292,616.642,560 hours; which, divided by 24, quotes 15.595,525.693,440 days; which makes 42,727.330,666 years, and 70 days. (3) As to the combinations, diſtinguiſht by the names of *choices*, and *compoſitions*, the doctrin of them is too abſtruſe, and ſo far out of the way of ordinary occaſions; that it would be improper to treat of them in a ſyſtem of this nature. The curious may be ſatisfied with what they will find on this ſubject in Malcolm, arithm. 5. 6. p. 514.

+ If the word *Philippa* were given, which conſiſts of 8 letters, of which (without conſidering thoſe which are of the ſame ſort) the changes will be 40320: but becauſe I is twice repeated, divide 40320 by 2, the changes on two letters; and the quote is 20160: and this, divided again, by 6 (the changes on 3, becauſe P is thrice repeated) gives in the quotient 3360; which are the changes in the word Philippa. Or, If I had divided 40320 by 12 (becauſe 2 times 6 is 12) the quotient would give, in the anſwer, at one operation, the ſame as before.

+ If two bells more had been added to the aforeſaid 12; they would have advanc'd the number of changes (and conſequently the time) beyond common belief. For 14 bells would require (at the ſame rate of ringing as before) about 16575 years to ring all their different changes.

§ For, if we ſuppoſe the words *bona*, and *mala*, continualy to keep the ſame (to-wit, the 10th) place, the reſt being 11 in number, (indifferently changing place with any other in the ſame verſe) the number of variations of 11 places will be 39916800: which, doubled (for the number of changes in both verſes) makes 79833600.

f *For example.*——Of 6 things, whereof 3 are the ſame, to find the changes: Say, the whole changes of 6 different things are 720; thoſe of 3 are 6: then, 720 ÷ 6 = 120.—Suppoſe 8 notes of muſic, whereof 3 are the ſame, and 2 are the ſame; but different from the former 3; and both different from the remaining 3 (as *fa*, *fa*, *fa*, *ſol*, *ſol*, *la*, *mi*, *fa*) What is the variety in the ſucceſſion of theſe 8 notes? (*Anſw.*) The changes of 8 different notes are 40320: of 2 there are 2, and of 3 there are 6: then 2 × 6 = 12; and 40320 ÷ 12 = 3360, the variety ſought.

g Thus, the choices OF 4 IN 9 are 126. For (1) 1 × 2 × 3 × 4 (the ſeries, *upwards*, to the number to be elected) multiplied continually together, = 24. (2) 9 × 8 × 7 × 6 (the ſeries, *downwards*, to the ſame number of terms, out of which the election is to be made) multiplied continualy together, = 3024. (3) Then, 3024 ÷ 24 = 126, the number of the choices of 4 in 9*.

a	b	c	a	d	e
a	b	d	b	c	d
a	b	e	b	c	e
a	c	d	b	d	e
a	c	e	c	d	e

a	b	c	d				g
			e			f	g
			f		e	f	g
			g	b	c	d	e
		d	e				f
			f				g
			g			e	f
		e	f				g
			g			f	g
		f	g		d	e	f
	c	d	e				g
			f			f	g
			g		e	f	g
		e	f	c	d	e	f
			g				g
		f	g			f	g
	d	e	f		e	f	g
				d	e	f	g

To take-out all the elections of any number out of a greater, the method may be conceivd from the following example.——The elections of 3 out of 5 things (a, b, c, d, e) are 10: the diſpoſition of which is as in the margin; and may be taken-out, thus: Firſt, I take-out the three firſt letters as they ſtand in order, a b c: then I put another in the firſt place (on the right) ſucceſſively in the order of the letters, till there is not another behind: then I put a new letter in the ſecond place (on the right) keeping it there till I change all the letters in order, that are in the firſt place: and, then, I put a new letter in the third place, keeping it there till I change all the letters in the ſecond place, as oft as poſſible; that is, ſo as there remain enow behind, to make out the number: and, with each of theſe, in the ſecond place, I change all thoſe in the firſt place: and ſo on, if there are more things elected; as may appear from the following example.——The elections of 4 in 7 things (a, b, c, d, e, f, g) are 35; as in the margin: the order of which, being carefully conſiderd, is ſufficient direction for any other caſe . . . Only it may be obſervd, that, when g comes in the firſt place (on the right) the letter, in the ſecond place, is chang'd; and there it ſtands, till g comes again in the firſt place, and then it is chang'd again; and ſo on, till f comes in the ſecond place: and then the letter, in the 3d place, is chang'd; and ſo it ſtands till the ſame changes, as before, fall upon the firſt and ſecond places; that is, till f g come together in the firſt and ſecond places: and then the letter, in the 3d place, is again chang'd: and ſo it goes on till e comes in the third place; and then the 4th place is chang'd; and ſo on, till ſuch a letter comes in the laſt place; that, what are behind in the order of the letters do juſt make-up the number elected.

* *NB.* (1) In any given number of quantities, the number of combinations increaſe, gradualy, till you come about the mean numbers; and ſo decreaſe, gradualy, again. So, in 8 quantities, there are more combinations of 3 and 5, than of 2 and 6; and more of 2 and 6, than of 1 and 7: as may be ſeen in the following table. (2) If, the number of quantities be even, $\frac{1}{2}$ the number of places ſhows the greateſt number of combinations, that can be made in thoſe quantities. So, if the number of quantities be 8; the $\frac{1}{2}$ of which is 4, ſhows the greateſt number of combinations in theſe quantities will be of 4 in 8, as in the table. But (3) If the number of the quantities be odd; then thoſe 2 numbers, which are next together, and whoſe ſum is equal to the given number of quantities; ſhow the greateſt number of combinations. So, of 7 quantities, the greateſt number of combinations will be of 3 and 4 quantities in 7; and are equal, as in the table.——*Example.* How many locks, whoſe wards differ, may be unlockt with a key of 8 ſeveral wards? (*Anſw.*) 255 locks: 8 whereof may have one ſingle ward; 28, double wards; 56, treble wards; 70, four wards; 56, five wards; 28, ſix wards; 8, ſeven wards; and 1 lock, eight wards: as appears by the upper table in the margin.

1	In 8 =	8
2		28
3		56
4		70
5		56
6		28
7		8
8		1

1	In 7 =	7
2		21
3		35
4		35
5		21
6		7

h And ſo on, *increaſing the powers*, according as your number of quantities increaſe.—*For example* (I) What number of chances are there on 2, 3, 4, 5, and 6 dice? (*Anſw.*) On 2 dice (by ſquaring the 6 ſides, will be found) 36 chances: on 3 (by cubing the ſides) 216: on 4 (by increaſing the powers) 1296: on 5, 7776: on 6, 46656. (II) What number of compoſitions of 3 letters in 20? (*Anſw.*) 8000; the cube of 20. (III) What number of combinations of 6 letters, in 24, or in the whole alphabet? (*Anſw.*) 191102976; the ſquard cube of 24. (IV) What number of hexameter and pentameter verſes, may be made out of the following lines?

HEXAMETER.

Turbida. ignea. peſſima. horrida. aſpera. martia. barbara. lurida. effera.

Fata. ſigna. damna. bella. vincla. ſiſtra. caſtra. ſcorta. tela.

Sequi. foris. pati. tuis. domi. patet. puto. palam. ferunt.

Præmonſtrant. proritant. promittunt. protendunt. producunt. monſtrabunt. progignent. prænarrant. promulgant.

Tempora. pocula. prælia. verbera. lumina. fœdera. agmina. crimina. ſidera.

Dura. ſæpe. quædam. acerba. prava. multa. dira. nigra. ſæva.

PENTAMETER.

Tetrica. ardua. perfida. improba. ſordida. impia. triſtia. turpia. noxia.

Præſtabunt. præſcribunt. concludunt. prædicunt. perficiunt. conſummant. conglomerant. ſignificant. procurant.

Dura. acta. vina. verba. dicta. facta. labra. arma. aſtra.

Doloſa. pudenda. proterva. nefanda. cruenta. ſuperba. moleſta. ſiniſtra. maligna.

Nova. aliis. tibi. viris. ſcio. mea. malis. vides. mihi.

NB. *Take any one word out of each line, and you will have a true verſe.*

(*Anſw.*) (1) In the Hexameters: there being 6 lines, and 9 words in every line; the ſquard cube of 9 gives 531441 * (2) In the Pentameters: there being 5 lines, and 9 words in every line; the ſurſolid, or 5th power of 9, gives 59049 verſes. (3) In All 590490: a number of verſes, that would make above 45 volumes as big as Virgil †. (V) What number of compoſitions may there be of the 24 letters of the alphabet, accounting them by 1 and 1, by 2 and 2, by 3 and 3, and ſo on to 24? (*Anſw.*) (1) If we account, each time, 24; the anſwer would be 1333735776850284124449081472843776. (2) But, if we are to find all the numbers preceding in geometrical progreſſion under it (according to the rule in progreſſion) the ſtating will be 23 : 24 : : 1333735776850284124449081472843775 : 1391724288887252-

9994251284934022200; which is the number of compoſitions ſought.

* So many verſes may be made out of the tables of hexameters, without taking notice of the permutation of places. For, you may change moſt of the 1ſt, and the 6th line, into the 5th, and 2d line.

† For 590490 (the number of verſes, that may be made by the words in the table above) ÷ 13016 (the number of verſes in Virgil) = 45 volumes, and 4771 verſes.—*NB.* To give this feat an air of myſtery . . . The author, John Peter (Sept. 29, 1677) diſtributed the letters of thoſe words into tables. (1) For the Hexameters there are 6 tables: each table has 10 cells a-croſs: and as many as are neceſſary to complete the words downwards. In theſe, the letters are ſo diſposd, that, if you take any of the firſt 9, and add every 9th till the word is completed; you will have the ſeveral words as in the tables; and, by that means, an hexameter verſe. (2) For the Pentameters, the method is the ſame . . . And, to ſtrengthen the paradox, he entitled the piece Artificial Verſifying; whereby any one of ordinary capacity, though he underſtands not one word of latin, may be taught immediatly to make 590490 hexameter and pentameter verſes, true latin, true verſe, and good ſenſe.

DIVISION.

1 DI′V[a]. (1) Say How-óft -ſor in -dénd[b];
or the 1ſt in the 1′ſt[c]. (2) By the
anſwer (*which is to be plac'd in the quotient*)
2 Múltiply-ſór; (3) and the próduct Subtract
from the -dend, by addition[d].
3 Thén, *for next ſtép*, advance 1′ in the -dénd;
and count-báck, i'th' remainder,
4 Só many ás are i'th' -ſór: There begín, as at
firſt; and ſay, How-oft, &c.[e].
5 *NB.* I′f by the 2d you Bórrow no móre than
you Pay by the 1ſt: Right[f].

COMPENDIUMS.

6 *Ciphers*, i'th' end of the -ſór; Set-off ſo-many places i'th' -dend; Reſt [g].

7 -ſor *ſingle Digits* Quote under; the Reſt to the next place prefixing [h].

COMPOUND DIVISION

8 of Different Denominations: ſee Practice.

a There are other ways of diviſion; 6 of which may be ſeen in Webſter, and 9 in Allingham. The following (of which I find no account in authors) is offerd as much the *ſhorteſt, and eaſieſt*; and, by the diſpoſition of the figures, the moſt commodious for operation, proof, valuation of remainders, &c. *. V. notes [d] [e].—*The Expedients for the dividing large ſums,* are by Logarithms, by Neper's bones (V. Multiplication, note [b]) and by Tabulating. V. Tabulating.

* *An example* will ſhew the difference: Here, in ſo ſhort a ſum, in the *common* way, there are no leſs(beſides dots, and daſhes) than 18 figures to write; in the *other*, only 4, or 5. Beſide the ſtraggling ſituation of the ſeveral members of the operation; more particularly of the diviſor, and quotient: which are at ſuch a diſtance, as render it inconvenient to multiply the one into the other.

The Common way.	*By my Rule.*
24) 1212289(50512	1212289(24
120	10241 50512
122	
120	
28	
24	
49	
48	
1	

b *For example:* To divide 36 by 12, ſay How-oft 12 (the Diviſor) in 36 (the Dividend): or 'The 12ves in 36'?—Anſw. 3; the Quotient, without any Remainder *.

For 3 × 12 = 36, which is the ſureſt method of *Proof*; to-wit, If the quotient, multiplied into the diviſor, plus the remainder, be equal to the dividend: but, for a ſhorter, V. Proof.

* Mr Hatton, in his mathematical manual (p. 183) among his numerical novelties, obſerves that the number 362880 may be continualy divided by a different digit, from 1 to 9 incluſive; without any *remainder* of any of the dividends: which cannot be ſaid of any other.

c *For example:* To divide 365 by 121, ſay How-oft 1 (the 1ſt of the divi *-ſor*) in 3 (the 1ſt of the divi *-dend*): or, rather (taking the two firſt) How-oft 12 in 36.—Anſwer, 3. V. note [d]. (*NB*) But here ariſes a *Difficulty*, to-wit, To hit-upon the proper anſwer. To ſettle this Point is the buſineſs of line 5.

d *For example:* To divide 365 by 121, ſay, as above (note [c]) The ones in 3—Anſw. 3; which ſet-down in the quotient, as in the margin. Then, by this anſwer, multiply 121 (the diviſor) and, at the ſame time, at each ſtep, ſubtract the product (without ſetting-it-down) from 365 (the dividend) Saying 3 × 1 = 3 + 2 † (to be ſet-down, as you ſpeak it, under the 5, *viz.* the 3d under the 3d) = 5. Then 3 × 2 = 6 + 0 (to be ſet-down under the 6, to-wit, the 2d under the 2d) = 6. Then 3 × 1 = 3 + 0 (as in the laſt) = 3. So, the Quotient is 3; the Remainder (which, in working with the pen, may be ſeparated, by a dot or comma, from the reſt of the remainder) 2: *i. e.* $\frac{2}{121}$. V. Fractions, and Rule-of-three, note [b] III, IV.

$$\begin{array}{ccc} \overbrace{365}^{\textit{Dividend}} & / & \overbrace{121}^{\textit{Diviſor}} \\ \underbrace{02}_{\textit{Remainder}} & & \underbrace{3}_{\textit{Quotient}} \end{array}$$

† Rather than 3 from 5, and there remains 2. V. Subtraction.—*NB.* The *advantage* of this method of ſubtraction is, that the laſt ſonnd in the mouth of the operator does more readily remind him of what he borrowd; and ſo ſecures againſt miſtakes in paying.

e *For example:* To divide 365365 by 121, ſay (1) for the firſt dividual, or partial dividend, How-oft 1 in 3, &c. as in note [d]. Then (2) for the 2d Step, advance one place in the dividend, to-wit, to 3; and count-back (in the remainder) ſo-many figures as

365365 / 121
02156 3019
106

there are places in the diviſor, to-wit, 3: ſo, the reckoning will end in 0: which (ſince you cannot have 1 in 0) enter in the quotient. Then (3) for the 3d Step, do as in the 2d; and the reckoning will end in 2: and There begin, as at firſt; and ſay How-oft 1 in 2, &c. Then (4) for the 4th Step, do as in the laſt; and the reckoning will end at 11: and There, again, begin, as at firſt; and ſay, as afore, How-oft 1 in 11, &c. V. note [f].

f *For example:* In the 4th ſtep of note [e], the queſtion is How-oft 1 in 11? anſwer 9 (more than which is never to be enterd into the quotient). Now, *To ſee whether it will go*; that is, whether it be not too-much; ſay (in operating by the 1ſt figure) $9 \times 1 = 9 - 11$ (remains) 2, to pay. Then (in operating the 2d figure) ſay $9 \times 2 = 18$: which, to ſubtract it from 6, obliges to borrow no more than 2. So, ſince, By the 2d, you borrow no more than you pay in the firſt, you are right; provided you have taken the firſt-leſs than what is too-much.

g *Thus,* Dividing (1) by 10, is cuting-off the laſt figure of the dividend. (2) by 20, is halving all but the laſt. (3) by 200, all but the two laſt, &c.—which places, thereby ſet-off, are to be conſiderd as the reſt, or remainder.

h *That is,* To divide by a ſingle digit, or any two ſuch as can be managd as one (e. g. 11, 12, &c.) without ſeting-down the diviſor, place the quotient under the dividend: and, for the manner and uſe of working in this way, V. Reduction, note [a], 1.

EQUATION.

1 Equ [a]. As the ſúms are to 1′: So the ſúms (into th' tímes) to the true time;
2 Whén what is due, at dífferent tímes, may (at ónce) be diſcharged [b].

a *Equation of Payments* is, when ſeveral debts are payable at ſeveral terms (bearing no intereſt till after the term of payment) to find a term, at which (if they are all payd) neither debtor nor creditor loſes any thing. The doing of this is calld equating the terms of payment; that is, reducing them to one.

For example: A is indebted to *B,* 100*l*; to be paid at the end of three months: alſo 200*l*; to be paid at the end of 4 months: and 300*l*; to be paid at the end of 5 months: Now, to prevent the trouble of many meetings, they agree to have but one payment of the three ſums, at one time: The queſtion is, when that muſt be, without loſs to either.——Anſw. As 600 (*l.* the ſums, or total of the debts) to 1: So 2600 (*l.* the ſums, each into their time) to 4 $\frac{1}{3}$ months: at the end of which time, if the 600*l* be payd, neither party will ſuſtain loſs.

O R

more accurately and unexceptionally
(according to Mr Malcolm, 6. 10)
as in the anſwer to the following queſtion:

There is 100*l*, payable 1 year hence; and 105*l*, payable 3 years hence: What is the equated time, allowing ſimple intereſt at 5 per cent per annum?

100*l.* the firſt debt:
105 with int. at 5*l.*
205 ÷ 5 (the int. = 41
+ (the ſum of times) 4
The (I) numb. found: 45

100*l.* the one debt:
× 1 its time =
100 the product.

105*l.* the other:
× 3 its time =
315 the product.

Then 100 + 315 = 415 ÷ 5 = 83 + 3 (the product of the two times) = 86 (the II number found). Then the ſquare of 45 (the I number) is 2025: of which the $\frac{1}{4}$ part is 506.25: from which taking 86 (the II number) the difference is 420.25: the ſquare root of which is 20.5.——This, ſubtracted * from 22.5 (the $\frac{1}{2}$ of number I) gives 2 (years) the true anſwer †.

* *NB.* This root, being *added to,* or *ſubtracted from* the $\frac{1}{2}$ of the I number; the ſum, or the difference, will be the time ſought——To know *which* is the anſwer; you muſt apply both, according to the conditions of the queſtion. Thus (1) If you take the *ſum*; then, if that is a time greater than the time to the laſt-payable debt; the difference will be the time ſought. Or (2) If you take the *difference*; and that be leſs than the time to the term of the firſt-payable debt; the ſum is the time ſought. But (3) Having tried either, and found it betwixt the terms

of payment of the two debts; you may try if the other does not caſt it beyond the laſt, or within the firſt term: for, in this caſe, that, which was firſt tried, is the anſwer. But (4) If both give times betwixt the two given terms; then you muſt examin which of them will make an equality of intereſt, and diſcount. V. Rebate.

† This is alſo provd by *application.* For this time (to-wit, 2) being exactly in the middle betwixt the two given times; the intereſt of 100 *l,* for 1 year, is equal to the diſcount of 105 *l,* for 1 year; each of them being 1 *l.*

NB. If there be *more debts than two*; find an equated time for the two, that are firſt payable. Then conſider their ſum as a debt payable at that equated time. And, find another equated time for that debt, and the next of the given debts. And, ſo on, through as many as are given.

EVOLUTION.

1 E'v. the reſólving a power ínto the párts
it's composd-of [a].
2 *Squáre* is the product óf any number ínto it
ſelf [b]: And
3 *Cúbe*'s the ſquare ínto th' ſame númber [c]:
The *Hígher powers* riſe in proportion [d].

4 *Square to Extract*

is to find the ſide of an area given.

For the MANNER *ſee Note* [e]: *For the* USE *ſee Note* [f].

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[a] When any number (calld, on this occaſion, the *root*) is drawn into it ſelf; and, afterwards, into that product, &c. it is ſaid to be ſo often *involvd* into it ſelf; and takes the name of a *Power*. Thus — 2 × 2 is the Square, or 2d power — 2 × 2 × 2 is the Cube, or 3d power — 2 × 2 × 2 × 2 is the Biquadrat, or 4th power — 2 × 2 × 2 × 2 × 2 is the Surſolid, or 5th power — &c.

[b] *Thus* 9 is the ſquare of the root 3: to-wit, 3 × 3 = 9.

Thus

c *Thus* 27 is the cube of the root 3 : to-wit, 3 × 3 = 9
d × 3 = 27.

The root, ſquare, and cube, take their names from geometrical extenſions. (1) A Root is repreſented by a line, or ſide ; having but one extenſion, to-wit, that of length only. (2) The Square is a plane, or figure of two dimenſions ; having equal length and breadth. (3) The Cube is a ſolid body of three dimenſions ; having equal length, breadth, and thickneſs—— But, beyond theſe three, nature proceeds not, as to local extenſion : that is, the nature of place or ſpace admits no room for other ways of extenſion, than length, breadth, and thickneſs : Neither is it poſſible to form, or compoſe any figure, or body, beyond that of a ſolid. And, therefore, all the ſuperior powers above the cube, or third power (as the Biquadrat, or fourth power ; the Surſolid, or fifth power, &c. *) are beſt explain'd and underſtood by a rank or ſeries of numbers in geometrical proportion. For inſtance, ſuppoſe any rank of geometrical proportionals, whoſe firſt term and ratio are the ſame : and to them let there be aſſignd a ſeries of numbers in arithmetical progreſſion, begining with an unit, or 1 ; whoſe common difference is 1 (V. Progreſſion) Thus :

1	2	3	4	5	6	7	*Exponts*
2	4	8	16	32	64	128	*in* ∺

Then are theſe numbers, in geometrical progreſſion, produc'd by a continued multiplication of the firſt term into it ſelf. And thoſe in arithmetical progreſſion (calld the indices, or exponents of the powers) ſhow what degree, or power, each term in the geometrical proportion is of. For example, in the above ſeries, 2 is both the firſt term or root, and common ratio of the ſeries. Then 2 × 2 = 4 the (ſecond term, or) ſquare : And 2 × 2 × 2 (or 4 × 2) = 8, the (third term, or) cube : &c.

* The *Superior powers* had particular names among the antients : but they are very complex and burthenſome to the memory ; and tend no way to the improvement, or eaſineſs of the ſcience : whereas it is obvious that we have no more to do, but diſtinguiſh them by their order in the ſeries of products, calling the firſt product the firſt power ; the ſecond product, the ſecond power ; and ſo c : whereby theſe names do, of themſelves, in a very ſimple and eaſy manner, diſtinguiſh the ſeveral powers, in conſe-

quence of the general definition of a power: for they expreſs the number of multiplications of the root in the production of each power; which the ancient names do not. —*NB.* On account of this conveniency it is, that, though the terms of root and powers, properly ſpeaking, ought always to be contradiſtinguiſht; ſo that the products only ſhould be calld powers: yet, here, the firſt root is alſo calld the firſt power; that the ſame figures might, both expreſs the number of the factor, and give denomination to the power.

The method of extracting the Square root is as follows:

Diviſors	Reſolvend	Roots
3	1027458916 /	32054
6	127	32054
640	34589	into it ſelf
6410	256416	gives
	00000	102745 &c.

Having pointed the reſolvend (begining with units) according to the number of places in the power [to-wit, 2, for the ſquare root; 3, for the cube; 4, for the biquadrat; &c]—Enter into your quotient 3 (the greateſt ſquare that is containd in the firſt period pointed-out to the left-hand) and multiply it into it ſelf (3 x 3) ſubtracting the product (to-wit, 9) from the period: the remainder will be 1 ——To this remainder (1) Bring-down the next period (to-wit, 27) (2) Then, doubling the quotient-root (to-wit, 3) for a diviſor, aſk how-oft 6 in 12, the unit-place being excluded. (3) The anſwer, 2, enter in the quotient: (4) And ſubtract the ſquare of it (to-wit, 2 x 2 =) 4 from 7: remains 3; which ſet-down underneath. (4) Then multiply it into the diviſor 6; and ſubtract it from the reſt of the reſolvend, as in common diviſion: remains 0.—To the remainder, (to-wit, 3) in like manner, (1) Bring-down the next period (to-wit, 45) (2) And, having doubled the quotient-roots (to-wit, 32) for a new diviſor (3) Aſk how-oft 64 (the doubled roots) in 34, the unit excluded, as before. (4) The anſwer, 0, enter in the quotient, and in the diviſor alſo. And—To the reſolvend (1) Bring-down the next period (to-wit, 16) (2) And aſk how-oft 640 in 3458 (all of the reſolvend but the unit's place) or 6 in 34: put the anſwer (to-wit, 5) in the quotient. (3) Then, ſubtract the ſquare of it (to-wit, 5 × 5 =) 25 from (9, borrowing 2, to-wit) 29: remains 4. (4) Then, multiply it into the diviſor (to-wit, 640) and ſubtract the products from the reſt of the reſolvend: the whole remainder will be 2564.——To this, again, bring-down the next period; and proceed as afore *.—Proof: 32054 × 32054 = 1027458916 †.

* *NB*. (1) When the reſolvend is not a true figurate number, according to the propoſd power (that is, is not a perfect ſquare, cube, &c.) ſomething will remain after the extraction has been made through all the points. Such numbers are calld *ſurd* numbers; and their roots can never be truly found: but will become a continued ſeries ad infinitum; if to the remainder there be ſtill annext ciphers according as the propoſd power requires, to-wit, by twos, in the ſquare; threes, in the cube; fours in the biquadrat, &c. and the operations continued-on as before. For example, if it were requird to extract the ſquare root of 6968; the laſt remainder will be 79; to which add 00; and proceed as by rule: the ſquare will come-out in decimals (by continuing to add ciphers) ,4745, &c. And, in this manner, the root of a ſurd number may be continued on to what exactneſs you pleaſe, but cannot be truly found. (2) What is here done in whole numbers, may be done (in like manner) in decimals, and mixt numbers: as alſo in vulgar fractions; by changing the given fractions into decimals.

† *For the Cube, or other Higher power,* the Method is the ſame; the differences being accommodated according to the nature of the power. Thus —[Rule] Every third figure, begining from unity, is firſt to be pointed; if the root to be extracted be a cubic one: or every fifth; it be a quadrato-cubic (or of the fifth power) and then, ſuch a figure is to be writ in the quotient, whoſe greateſt power (that is, whoſe cube, if it be a cubic power; or whoſe quadrato-cube, if it be the fifth power, &c.) ſhall either be equal to the figure or figures, before the firſt point; or next leſs under them. And then, having ſubtracted that power, the next figure will be found by dividing the remainder (augmented with the next figure of the reſolvend) by the next leaſt power of the quotient multiplied into the index of the power to be extracted; that is, into the triple ſquare, if the root be a cubic one: or into the quintuple biquadrat (that is, five times the biquadrat) if

the root be of the fifth power, &c. And, having again ſubtracted the power of the whole quotient from the firſt reſolvend, the third figure will be found by dividing that remainder (augmented by the next figure of the reſolvend) by the next leaſt power of the whole quotient, multiplied by the index of the power to be extracted. Thus [Example] To extract the cube root of 13312053, the number is firſt to be pointed after this manner, to-wit, 13̇312̇053̇ (to-wit, at unity, and every third figure from it) then, you are to write the figure 2, whoſe cube is 8, in the firſt place of the quotient; as, that which is the next leaſt cube to the figures 13 (which is not a perfect cube number) or as far as the firſt point; and, having ſubtracted that cube, there will remain 5; which being augmented by the next figure of the reſolvend 3, and divided by the triple ſquare of the quotient 2, by ſeeking how many times 3 × 4, or 12 is contain'd in 53; it gives 4, for the ſecond figure of the quotient. But, ſince the cube of the quotient 24, to-wit, 13824, would come-out too great to be ſubtracted from the figures 13312, that precede the ſecond point; there muſt only 3 be writ in the quotient. Then the quotient 23 being in a ſeparate place multiplied by 23, gives the ſquare 529; which (again) multiplied by 23, gives the cube 12167: and this, taken from 13312, will leave 1145; which augmented by the next figure of the reſolvend 0, and divided by the triple ſquare of the quotient 23 (to-wit) by ſeeking how many times 3 × 529 or 1587 is contain'd in 11450; it gives 7, for the third figure of the quotient. Then the quotient 237, multiplied by 237, gives the ſquare 56167; which again, multiplied by 237, gives the cube 13312053; and this, taken from the reſolvend, leaves 0. Whence it is evident that the root ſought is 237.

		13̇312̇053̇	(2 3 7
Subtract	cube	8	
12)	rem.	53	(4, or 3
Subtract	cube	12167	
1587)	rem.	11450	(. . . 7
		13312053	
	rem.	 0	

f *The uſe* of this rule is very conſiderable in many branches of geometry, and other parts of the mathematics: but, becauſe the operations, that ſhow the uſe of it, would be unintelligible without a knowlege of the principles of thoſe ſciences; it would be abſurd to exemplify the application of them here. V. Part II. pref.

EXCHANGE.

1 Exchange, the barter of Coins, according to Par [a], or to Courſe [b], thus:
2 Coin, by reduction, *value* [c]: *exchange*, by diviſion i'th' ſame name [d];
3 Or, by the rule-of-three, when it can't well be dóne by diviſion [e].

a *Par of exchange* is what one nation *ſhould* allow another in exchange; which is certain and fixt, according to the intrinſic value of the coins of each.

b *Courſe of exchange* is what one nation *will* allow another in exchange; which is uncertain and contingent, ſometimes more, ſometimes leſs; according as mony is plenty or ſcarce, or according to the time allowd for payment, &c.——For the ſhowing the *exchange-value by inſpection*, volumes of printed tables calculated at the uſual rates, are in almoſt every merchant's counting-houſe.

——The uſual exchange is, between *London* and (1) *France:* in Crowns. (2) *Germany:* in Florins. (3) *Italy:* in Ducats, or Pieces of 8. (4) *Netherlands:* in Pundts. (5) *Portugal:* in Millreas. (6) *Spain:* in Rials and Pieces of 8. . . . For the value of which, &c. ſee the Tables, note [e].

c *For example:* To *value* 118 crowns, 1 livre, 11 ſous, 1 denier, and $\frac{1}{3}$; multiply the crowns into the number of livres that make a crown, to-wit, 3; adding the 1 livre, and ſo on, as in common reduction, to the loweſt name; to-wit, thirds of a denier: the product will be 256000: which divided by 40 (the number of 3ds of a denier, that make 1 penny engliſh) gives 6400 *d*; that is (by reduction upwards) £ 26 13 4.

d *For example:* ——How-much french mony muſt I have in *exchange* for 40 marks? (Anſw.) A mark is 13 *s* 4 *d*; tha that is, in the loweſt name, 160*d*. 40 marks, therefore, are (40 × 160) 6400 *d*: which divided by a french crown (to-wit, 4*s* 6*d*) in the ſame name (to-wit, 54*d*) gives 118 crowns: remains 28; that is, in the next lower name (to-wit, livres) 1 livre. for, 3 (the number of livres in a crown) × 28 = 84 ÷ 54 = $1\frac{30}{54}$ livres. Then 30 (the remainder) × 20 (the number of ſous in a livre) = 600 ÷ 54 = $11\frac{6}{54}$ ſous. Then 6 (the remainder) × 12 (the number of deniers in a ſou) = 72 ÷ 54 = $1\frac{18}{54}$ or $\frac{1}{3}$ denier. And ſo-much muſt I have for 40 marks— I would have, in exchange for 309*l* 8*s*, an equal number of crowns and dollars; the crowns, at 5*s* 6*d*; the dollars, at 4*s* 5*d*. (Anſw.) 309*l* 8*s* = 74256*d*. And 5*s* 6*d* × 4*s* 5*d* = 119*d*. Then 74256 ÷ 119 = 624, the number of crowns and dollars I am to have.——A Bill of exchange * was accepted at London, for the payment of 400*l* ſterling, for the like value deliverd in Amſterdam at 1*l* 13*s* 6*d*, for 1*l* ſterling. How-much mony was deliverd at Amſterdam? Anſw. 670*l* flemiſh. For, 1*l* = 240*d*; and 1*l* 13*s* 6*d* = 402*d*. Then 240 : 402 :: 400 : 670, the anſwer requir'd. V. Practiſe.

* *The Form of a bill of exchange:* Rotterdam, April 10, 1746, for £ 1272 13 4 flemiſh, at 33*s* 4*d* per pound ſterling. 'At uſance, pay this my firſt of exchange, ſecond Mr. Edward Jones, or his order, twelve hundred ſeventy-two pounds, thirteen ſhillings, and four pence flemiſh: exchange, at thirty-three ſhillings and four pence per pound ſterling; for the value receiv'd of Mr. John Hall: and account it to your humble ſervant, Herman Vanderſtagen. —Here Hall pays the mony in Holland (and is calld the *remitter*) to Vanderſtagen (who is the *drawer*) drawing his bill on the Varelſts, (his correſpondents at London) to pay the value to Jones at London, who is Hall's *correſpondent*.' To Meſſ. *A* and *B* Varelſt, merchants in London.

As it generaly happens in Queſtions (I) relating ſimply to the *par* of exchange, (II) wherein *gain and loſs*, and the *allowances to factors*, are conſiderd, (III) or that relate to the *arbitration of exchange*.

I. *In queſtions relating ſimply to the* PAR *of exchange.* ——1. If the proportion is given betwixt two ſpecies directly; the ſolution is by one operation of the rule of three. For example: A merchant, at Amſterdam, paid

150 guilders for 13 *l* 15 *s* ſterling, receivd by his correſpondent at London: (1) What is the rate of exchange? or what is 1 guilder valued-at in engliſh mony? . . . Say, If 150 guilders give 13 *l* 15 *s*; what will one guilder? Anſw. 22 *d*. (2) What is one pound ſterling valued-at in dutch mony? . . . Say, if 13 *l* 15 *s* give 150 guilders; what will 1 *l*? Anſw. $10\frac{10}{11}$ guild.—— 2. When the proportion is given between each of the two ſpecies in queſtion, and a third ſpecies: the ſolution is by two operations of the rule of three. For example, I would exchange 200 ducats, worth 7 *s* a-piece, for dollars at 4 *s* 8 *d* a-piece. (1) How many dollars ought I to have? . . . Say, If one ducat give 7 *s*? what 200? Anſw. 1400 *s*. Then, If 4 *s* 8 *d* give one dollar, how many will 1400 *s*? (2) If 4 *s* 8 *d* give 1 dollar, how many will 7 *s*? Anſw. $1\frac{1}{2}$. ſo that one ducat is worth $1\frac{1}{2}$ doll. . . . Then, ſay, If 1 duc. give $1\frac{1}{2}$ dollar; how many will 200 give? (3) Had it only been requird to find the direct proportion betwixt crowns and dollars: then, ſay, If 7 *s* give 1 ducat; how many will 4 *s* 8 *d*? Anſw. $\frac{2}{3}$ of a ducat . . . Or, ſay, If 4 *s* 8 *d* give one dollar; how many will 7 *s*? Anſw. $1\frac{1}{2}$ dollar.—— 3. When the proportion is given betwixt one ſpecies and another; betwixt this other and a third; betwixt this third and a fourth; and ſo on; as far as you pleaſe to find the exchange betwixt the firſt and laſt ſpecies: the ſolution is by one fewer operations of the rule of three than there are different ſpecies; in manner of the following queſtion: *(For example)* Exchange betwixt London and Amſterdam being at 1 *l* ſterling for 38 *s* flemiſh; betwixt Amſterdam and Francfort, at 6 *s* flemiſh for 66 cruitzers; betwixt Francfort and Paris at 54 cruitzers for one crown: what is the exchange, according to that courſe, betwixt London and Paris?—Anſw. 1 *l* for $7\frac{20}{27}$ crowns; found thus: Set-down the given terms, as in the margin; and, working from the left-hand to the right, Say: If 6 *s* give 66 cru. what will 38 *s*? the anſwer is 418 cru. which is ſet under Francf. Then, ſay: If 54 cru. give 1 crown; what 418 cru.? the anſwer is $7\frac{20}{27}$ cr. which

Lond.	Amſt.	Francf.	Paris
1 *l* =	38 *s*		
	6 *s* =	66 cru.	
		54 cru. =	1 cr.
1 *l* =	38 *s* =	418 cru. =	$7\frac{20}{27}$ cr.

which is ſet under Paris. And, thus, the exchange betwixt London and Paris is found to be at 1 *l*; for $7\frac{20}{27}$ crowns. (*NB*) (1) The undermoſt line ſhows not only the exchange betwixt the firſt and laſt places; but alſo betwixt any two of them: the quantities, in that line, being evidently all equal in value, from the nature of the operation. (2) If there are two different given ſpecies for one place, they muſt be reduced to one ſpecies. So, if the exchange betwixt Amſterdam and Francfort, were expreſt by 36 ſtuyvers for 66 cruitzers; then we muſt firſt reduce the 38 *s* to ſtuyvers, or the 36 ſtuyvers to ſhillings, by the known relation of ſtuyvers and ſhillings; which is 6 ſtuyvers to one ſhilling. Or, if it were 36 ſtuyvers for 1 florin; then you muſt alſo reduce the 1 florin to cruitzers, or 66 cruitzers to florins. And, if this reduction cannot be done; that is, the relation of theſe ſpecies is not known, the queſtion cannot be ſolvd. (3) If there is another given quantity of the firſt and laſt place; to find a quantity of equal value in the other, it is a plain application of the rule of three, from the rate of exchange found as was obſerv'd upon the preceding queſtion. (4) All theſe operations of the rule of three may be reduc'd to one diviſion, thus: multiply the conſequents of all the proportions (that is the firſt number under every place) for a dividend; and all the antecedents (or ſecond numbers under every place) except the firſt, for a diviſor: the quotient will be the number ſought of the ſpecies of the laſt place, equal to the number under the firſt place. Thus, in the preceding example, 38 multiplied into 66, produces 2508 [the number of crowns, being 1, does not multiply] Then 54, into 6, produces 324: and 2508, divided by 324, produces $7\frac{20}{27}$ *.

* The reaſon of this will be manifeſt, by conſidering how the ſeveral operations of the rule of three are made. For, the anſwer of the firſt operation is the quote of 38 multiplied into 66, and the product divided by 6: which we may expreſs thus: $\frac{38 \times 66}{6}$. Then, for the ſecond operation, it is the laſt anſwer multiplied into 1 (which is ſtill $\frac{38 \times 66}{6}$) and the product, divided by 54: which is $\frac{38 \times 66}{6 \times 54}$; according to the direction now given. And,

how many places ſo-ever there be, the reaſon is manifeſtly the ſame. V. Proportion.

II. *In queſtions wherein* GAIN and Loſs, *and the* Allowances to FACTORS *are conſiderd*; the ſolution is as follows: *For example*——*A*, of London, draws upon *B*, of Amſterdam, 500 guilders at 22 *d* per guilder: for which *B* redraws upon *A* at 21 *d* per guilder, with proviſion at $\frac{1}{2}$ per cent, and ten guilders of brokerage. (1) How much will *A* pay? and (2) Whether has he gaind or loſt? (Anſw.) Say, As 1 guilder to 22*d*: ſo is 500 guilders to 11000 *d*, receivd by *A* for the draught: Then, As 100 to $\frac{1}{2}$; ſo 500 to $2\frac{1}{2}$, the proviſion due to *B*; which, with 10 guilders brokerage, added to 500, makes $512\frac{1}{2}$: for which he draws on *A*. Therefore, Say: As 1 guilder to $21\frac{1}{2}d$: ſo $512\frac{1}{2}$ to $11018\frac{3}{4}$. So *A* (1) pays, for the redraught, $11018\frac{3}{4}d$; and (2) loſes $18\frac{3}{4}d$. (*NB*) (1) If *A* had remited to *B*, with orders to remit the value back again: then, having found what *B* receives by *A*'s remitance: from that ſubtract *B*'s proviſion, and brokerage; the remainder is what he remits to *A*: whoſe value being found, the compariſon of that, and of what *A* paid for the remitance, ſhows what he gains or loſes. (2) If *A* draws upon *B*, and afterwards remits the value to him; he muſt add the proviſion, due to *B*, to the Sum which *B* paid; and remit the total. (3) If *B*, by *A*'s order, draw-upon him, and afterwards remits him the value; then *B* deducts his proviſion, and double brokerage, from what he receivd by the draught; and remits the remainder. (4) In all the caſes, where *A* draws, or remits; you muſt alſo conſider what brokerage it may have coſt him, in order to know his gain or loſs.——If exchange from London to Amſterdam is at 1 *s* 10 *d* for 1 guilder; and to Paris, at 3 *s* 8 *d* for 1 crown: alſo, from Amſterdam to Paris, at 40 ſtuyvers for 1 crown: Whether is it more profitable that London remit directly to Paris, or by way of Amſterdam? that is, remit to Amſterdam, to be remited from that to Paris, (Anſw.) Find (by the rule of conjunct proportion) what 1 *l* ſterling is worth at Paris, according to the courſe with Amſterdam; and, alſo, according to the exchange directly with Paris: the compariſon of theſe values of 1 *l* gives the anſwer. (*NB*) (1) What allowance is due to the factor at Amſterdam is to be deducted from the mony he receives; that is, the value of 1 *l* ſterling in guilders: and the remainder is what he remits to Paris. (2) If there are more than 3 places in the queſtion; and the exchange is given betwixt one and another in a ſeries, and alſo betwixt the firſt and laſt: to find which is moſt profitable, to remit from the firſt to the laſt place directly, or through all theſe places: you muſt find what is the exchange between the firſt and laſt place, according to the courſes through all the other places; and compare that with the given exchange betwixt the firſt and laſt place.

III. *In queſtions relating to what is call'd the* ARBITRATION *of Exchange*, the Solution is as follows: *For example*——*A*, of Rochel orders *B* of Amſterdam to draw upon him at $97\frac{3}{4}$ ſols for 1 crown, and to remit the ſame to Hamburg at 34 ſols, for 1 dollar. *B*, cannot draw, but at 97 ſols for 1 crown, how ought he to remit to follow his order? (Anſw.) at $33\frac{17}{25}$ ſols for 1 dollar: Found thus: as $97\frac{3}{4}$ to 97, ſo 34 to $33\frac{17}{25}$. For, the courſe being below the order in the draught to Rochel, it ought to be ſo proportionaly in the remittance to Hamburg . . . Or, the reaſon of the work may be conceivd thus: If, for 1 cro. which *A* pays at Rochel, he gets only 97 ſols at Amſterdam, by the courſe; inſtead of $97\frac{3}{4}$ (which was his order) then, for 1 dollar, he receives at Hamburg, he ought to pay proportionably fewer ſols than 34; which was his order.——*A*, of Amſterdam, orders *B* of Amſterdam to remit to Rouen, at $103\frac{1}{2}$*d* flemiſh, for 1 crown; and to draw the ſame on him at 225 gros for 1 *l* flemiſh: *B* cannot draw under 230 gros. How ought he to remit, to follow his order? (Anſw.) At $101\frac{1}{2}$*d* flemiſh for one crown; Found thus: as 230 to 225: ſo is $103\frac{1}{2}$ to $101\frac{1}{4}$. For, if *A* pays at Coningſberg, more gros than order (for 1 *l* receiv'd at Amſterdam) he ought to receive, proportionaly, more crowns than order at Rouen (for $103\frac{1}{2}$*d* paid at Amſterdam): or, which is the ſame thing, he ought to pay proportionaly leſs than order, at Amſterdam, for 1 crown receivd at Rouen.

Coning.	Amſt.	Rou.
	d	cro.
groſs	$103\frac{1}{2}$ =	1
225 =	1 *l*	
330 =	1 *l*	

FALSE

FALSE.

1 FALSE [a]: Suppoſitions and Errors multiply croſs-ways: the Products,
2 added, divide by the Sum of the errors;
if ſupps more AND *leſs* [b]: But,
3 *If more* OR *leſs*; ſubtract, and the Difference
by th' difference of err. div [c].
4 *If only one ſuppoſition*; the truth may be
found by proportion [d].

a When a queſtion cannot readily be applied to the rule of three, or any of the vulgar rules in arithmetic: the beſt expedient for ſuch as are not acquainted with algebra, is the rule of falſe (as it is commonly calld) which teaches us, by falſe terms ſuppos'd, to diſcover the true terms requird.——*By way of diſtinction* this rule may be divided into that of ſingle, or double poſition. (1) The rule of *ſingle* poſition is, when (at once, *viz.*) by one falſe poſition, we have means to diſcover the true reſolution of the propoſd queſtion. V. note [d]. (2) The rule of *double* poſition is, when two falſe poſitions are ſuppoſd for the ſolution of the queſtion propounded. V. note [b], &c.——*To know whether a queſtion be reſolvable by the rule of falſe, or not* (1) When a queſtion requires ſome given number to be divided by the number ſought, or any part of it: alſo when the number ſought, or ſome part thereof is to be ſquar'd, cub'd, &c: likewiſe, when ſome parts of the number ſought are to be multiplied one by the other: the ordinary rule of falſe will be uſeleſs. For example: 'What number is that, by which if 360 be divided, the quotient will be 24? Here, if two poſitions, or feign'd numbers, be taken; and 360 be divided by each of them: the errors will not be in the ſame proportion with the differences between the two numbers ſought and the two feignd numbers: and therefore the rule of falſe will be usd in vain. Yet, if it be aſkt 'What number is that, which being multiplied by 24, the product will be 360': the anſwer to this latter queſtion is the ſame with the anſwer to the former, and may be found by the rule of falſe. (2) But, when the queſtion requires the number ſought to be increaſt, leſſend, multiplied, or divided by ſome given number in any of thoſe caſes: ſuch analogy will ariſe as will give the true anſwer in the rule of falſe; to-wit, when the firſt error has ſuch proportion to the ſecond, as the difference between the number ſought and the firſt feignd number. Obſerve, therefore, what operation the queſtion requires to be performd with the number ſought, and ſome given number, or numbers: the ſame kind of operation, in every reſpect is to be made with each of the two feignd numbers (commonly calld poſitions) and the ſaid given number or numbers, which threefold proceſs being finiſht (whether it be by any one, or all of theſe rules, to-wit, addition, ſubtraction, multiplication and diviſion) there will ariſe three remarkable numbers or reſults; to-wit, one proceeding from the true number ſought; and two others reſulting from the two feign'd numbers: Then, from theſe three reſults the errors are collected; which are nothing elſe but the differences between the true reſult, and each of the two falſe reſults.

For example: Admit a church has a chancel 40 foot long; and that the ground, taken-up by the belfrey, is $\frac{1}{4}$th of the chancel, and $\frac{1}{6}$ of the nave; the nave 3 times the length of the belfrey, and $\frac{3}{4}$ of the chancel. How long is the whole church, within the walls; and every part of it? (Anſw.)——Suppoſe the nave of the church 150 feet (1) $\frac{1}{6}$th of that is 25: $\frac{1}{4}$th of the chancel, 10: in all 35, for the belfrey. (2) Then the nave, according to the queſtion, is 3 times that, to-wit, 105; and $\frac{3}{4}$ths of the chancel, to-wit, 30: in all 135. (3) Which being leſs than 150 ſuppos'd, the error is 15.——Suppoſing the nave be 102. (1) $\frac{1}{6}$th of that, is 17: $\frac{1}{4}$ of the chancel, is 10: in all 27 for the belfrey. (2) Then, 3 times that belfrey, is 81: and $\frac{3}{4}$ths of the chancel, is 30: in all 111. (3) which being more than the 102 ſuppoſd, the error is 9——Now, (1) The product (croſs-ways) of 150 (the 1ſt ſuppoſition) into 9 (the 2d error) is 1350: and of 102 (the 2d ſuppoſition) into 15 (the 1ſt error) is 1530. (2) Theſe added together (the ſuppoſitions being, the one, more; the other, leſs) make 2880. (3) which divided by 24 (the ſum of the errors) give 120 for the length of the nave. (4) And, the chancel being 40; and the belfrey ($\frac{1}{6}$th of 120, and $\frac{1}{4}$th of 40; that is) 30: the length of the whole church will be 190.

c *For example:* Three merchants built a ſhip, which coſt 1600*l.* *A* paid a ſum not-known: *B* paid double to *A*, within 50*l.* *C* paid as much as *A* and *B*, wanting 100*l.* What did each pay of the coſt? (Anſw.)——Suppoſing *A* paid 200*l*; *B* muſt have paid 35; and *C* 450: in all 1000*l*; leſs than the coſt (to-wit, 1600) by 600, for the 1ſt error.——Suppoſing *A* paid 250*l*; *B* muſt have paid 450; aud *C*, 600: in all 1300; leſs alſo than the coſt by 300.——Now, (1) The product of 200 (the 1ſt ſuppoſition) by 300 (the 2d error) is 60000: and 250 (the 2d ſuppoſition) by 600 (the 1ſt error) is 150000. (2) Then (the ſuppoſitions being both leſs than the truth) ſubtract the one from the other; the difference will be 90000. (3) which divided by 300 (the difference of the errors) gives 300*l* for *A*: conſequently 550, for *B*; and 750, for *C*: which make-up the coſt, to-wit, 1600*l.*

d *For example:* Three men built a houſe, which coſt 300*l.* *A* paid a ſum unknown: *B* twice as much: *C* 3 times as much. What did each pay of the 300? (Anſw.)——Suppoſe *A* paid 40*l* then *B* muſt pay 80; and *C*, 120: in all 240.——Then ſay (240 : 40 : : 300 : 50) If 240 ariſes from ſuppoſing 40: from 300 will ariſe 50 for *A*: conſequently, for *B*, 100; for *C*, 150: in all 300.

FELLOWSHIP.

FELL. for the keeping accounts in párt-
nerſhip; wíth, or without time.

1 A's Stocks of áll to the *gain*: ſo each man's
Stock to his *ſhare* [a]: Or

2 Gain by ſtóck; into th' quótient, each
ſtock [b]——*If with time*; into th' time,
ſtocks [c],

3 *Stócks to find:* A's gain to ſtocks; ſo each
man's gáin to his ſtock is [d].

Three

a Three partners make a joint-ſtock. *A* puts-in 5 *l*, *B* 10 *l*, *C* 20 *l*. With this they trade; and gain (or loſe) 12 *l*. What is each partner's ſhare?——Anſw. As 35 (the whole ſtock of all the partners) to 12 (*l.* the gain or loſs) ſo is 5 (*l*, *A*'s ſtock) to his ſhare of the gain or loſs, to-wit, *l* 1 14 3 $1\frac{5}{7}$.——And, in like manner, may be found the ſhares of the other two partners: which, if they make-up the whole gain or loſs, ſhow the work to be right. But,

b To anſwer queſtions of this nature, *the readieſt way*, is (decimaly) to divide the gain by the ſtock; and, into the quotient, to multiply each partner's ſtock.——Thus (to ſolve the preceding queſtion) 12,0000 (the gain) ÷ 35 (the ſtock) = ,3428: and this ,3428 × 5 (*A*'s ſtock) = 1,7140: that is £ 1 14 3 2 nearly——And ſo of the reſt.

c *If the ſtocks are in trade for different Times* [as, Suppoſe *A* and *B* join in partnerſhip upon theſe terms, to-wit, *A* agrees to lay-down 100 *l*; and to employ it, in trade, 3 months. Then *B* is to lay-down his 100 *l*: and, with the whole ſtock of 200 *l*, they are to trade 3 months more. Now, at the end of that time, they find their gain to be 21 *l*. What, then, is each man's ſhare of the gain according to his ſtock, and the time of employing it?]——The anſwer is found as afore, with this difference only, that each man's ſtock is to be multiplied into the time it is in trade——Say, then, As 900 (*l*, the whole ſtock, each part being multiplied into its time) to 21 (*l*, the gain) So is 600 (*l*, *A*'s ſtock multiplied into its time) to *A*'s ſhare, to-wit, 14 *l*.

d *For example:* Three partners make a joint-ſtock of 35 *l*; with which they trade, and gain 12 *l*: of which £ 1 14 3 $1\frac{5}{7}$ comes to *A*'s ſhare. What was his ſtock? ——Anſw. As 12 *l* (the whole gain) to 35 *l* (the whole ſtock) ſo £ 1 14 3 $1\frac{5}{7}$ (*A*'s gain) to 5 *l* (*A*'s ſtock)—— And, in like manner, the gain of the other two being given, their ſtock may be found by the ſame proportion, V. note, [a].

e *In ſuch queſtions where each perſon's ſtock Varies by addition and ſubtraction*, we muſt conſider how-long each part of the varying ſtocks continued in company; and, multiplying them by their times, the ſums of theſe products are the numbers, by which the diviſion is to be made.

—*For example*: Suppoſe *A* put in 40*l*; and, at 4 month's end, took-out 10*l*; and, at 2 months thereafter, put-in 30*l*: *B* put-in 50; and in 3 months, put-in 20*l*; and, at 8 month's end, they balance their accounts; and find 18*l* gaind: what is the ſhare of each? . . . Here, we muſt conſider how long each part of the varying ſtocks continued: and, multiplying them by their times, the ſums of theſe products are the numbers, by which the diviſion is to be made; as follows:

A had 40*l*	then 30*l*	then 60*l*	*B* had 50*l*	then 70*l*
for 4 m.	2 m.	2 m.	3 m.	5 m.
160	60	120	150	350

The ſum of *A*'s ſeveral products (to-wit, 160 + 60 + 120) is 340: of *B*'s (to-wit, 150 + 350) is 500 . . Then 340 + 500 = 840 . . And 840: 18: : 340, and 500, ſeveraly, to the ſhares of 18 ſought.—*A Complex queſtion* under this head (to give a ſpecimen of unraveling an intricacy) demands here an eclairciſſement. (Queſt.) *A*, *B*, and *C* keep company: *A* put-in, the 1ſt of march, 60*l*. *B* put-in, the 1ſt of may, 160 yards of broad-cloth: and *C* put-in, the 1ſt of june, 240 ducats. On the firſt of january following they accounted their gain; of which *A*, and *B* took-up 456*l*; *B* and *C* took-up 431*l*; and *C* and *A* took-up 375*l*. The queſtion is, What was gaind as well in the whole, as a-part? What *B* valued a yard of cloth at? and What were *C*'s ducats per piece? (Anſw.)——To find the Gains. (I) for the Whole gain: 456 (*A* and *B*'s gain) + 431 (*B* and *C*'s gain) + 375 (*C* and *A*'s gain) = 1262: the half of which (every man's mony being nam'd twice) to-wit, 631, is the whole gain. (II) For the ſeveral gains: (1) *A*, 631 (the whole gain)—431 (*B* and *C*'s gain) = 200, *A*'s gain (2) *B*, 631 (the whole gain) --375 (*C* and *A*'s gain) = 256, *B*'s gain. (3) *C*'s gain, therefore (to make-up the whole gain) muſt be 175*l*. ——To find the value of a yard of *B*'s broad-cloath. (1) Firſt ſay, If 200*l* (*A*'s gain) come from 60*l* (*A*'s ſtock) what will 256*l* (*B*'s gain) come from. Anſw. 76.8*l*. (2) Then, ſay If 10 months (*A*'s time) come from 76.8*l* (the anſwer to the former ſtating) what will 8 months (*B*'s time) come from. Anſw. (by inverſe proportion, leſs time requiring a greater ſtock) 96*l*, the value of the whole cloth. (3) Then, If 96 *l* be the value of 160 yards; What is the value of 1 yard? Anſw. .6 = 12 *s*. —To find the value of *C*'s ducat (after the ſame method) (1) Firſt 200 : 60 : : 175 : 52.5 *l*. (2) Then 10 : 52.5 : : 7 : 75 *l* = 240 ducats. (3) Then 240 *duc.* : 75 *l* : : 1 *duc.* .3125 *l* = 6 *s* 3 *d*.—Of the like nature with the foregoing are ſuch queſtions as this: Three perſons, *A*, *B*, *C*, hire (together) certain paſture-ground, for 24 *l*: in which *A* keeps 40 cows, for 4 months; *B* keeps 30 cows, for 2 months; and *C* keeps 36, for 5 months. How-much of the rent ought each of them to pay? (Anſw.) Multiply each perſon's number of cows by the time they were kept; and, by theſe products, proportion the rent. And, if the partners take-out, and put-in cattle, at different times; then work as in the preceding queſtion.

In caſes of Factorſhip, ariſe ſuch queſtions as theſe.—I. A merchant delivers to his factor 100 *l*; allowing him to join to it 30 *l*: and values his ſervice worth 40 *l*. What ſhare of the gain ought the factor to have? For the (Anſw.) ſubtract 40 from 100, which leaves 60; and proportion the ſhares of gain to 60 (for the merchant) and 70 (that is, 40 and 30) for the factor.——II. A merchant's real ſtock being 100 *l*, and the factor's 30 *l*; who receivd $\frac{1}{3}$ of the gain? What was his ſervice valued at? (Anſw.) Find the 3d part of 130; from which take 30: the remainder is the anſwer.——III. A merchant's real ſtock being 100 *l*, and the factor's ſervice valu'd at 20 *l*; who receivd $\frac{1}{2}$ of the gain; What was the factor's real ſtock? (Anſw.) 60; becauſe 20 and 60 make 80, the half of 160, the total real ſtock.——IV. The merchant's real ſtock being 100 *l*, and the factor being allowd $\frac{1}{4}$ of the gain for his ſervice: What real ſtock muſt he join to have $\frac{1}{3}$ of the gain? (Anſw.) When the factor gets $\frac{1}{4}$ (without any real ſtock) his ſervice is there valued at 25 *l*. With this value of his ſervice proceed to find the real ſtock, that he muſt have, to get $\frac{1}{3}$ of the gain by the methods of queſtion 3. (Note) In the preceding queſtions we may ſuppoſe 2, or more merchants, with the factor: it will be eaſy to apply the ſame rules, by adding the ſtocks of all the merchants into one ſum; and conſidering that, as one ſtock. And then, beſides what is already demanded, it may alſo be demanded to find the gain of each merchant. Thus, by the firſt method of an imaginary ſtock, what remains to the merchants after the factor's part is deducted, muſt be divided in proportion to their real ſtocks. By the real ſtocks we muſt divide the factor's eſtimation into parts proportiond to the merchant's real ſtocks; and take the parts anſwering to each from itſelf: the remainders are the numbers by which the merchants ſhares are to be proportiond.——V. A merchant's real ſtock being 120*l*, and the factor's 60: they agreed, that, at a year's end, the factor ſhould have ½ of both the ſtocks and gain: but they broke-up at 8 months end, having gain'd 150*l*. How-much ought the factor to have? (Anſw.) Here, it is plain the factor, for 12 months ſervice, was to have not only the gain of 30*l*, of the merchant's ſtock; but alſo 30 of the ſtock itſelf. So that his ſervice was valued at 30*l* real ſtock; but, the ſociety laſting only 8 months; it is plain he ought only to have 20*l* (which is to 30*l*. as 8 months to 12) And this, added to his own 60, makes 80*l*: which he receives of the real ſtock: and the merchant's part being 100*l*; then it is as plain that the 150*l* gain muſt be divided into 2 parts, proportiond to theſe ſtocks 80, and 100.

In other caſes relating to Stocks, Gain, &c. there occur other queſtions; for the ſolution of which it may not be improper to give ſome more particular hints: as——I. *A*, *B*, and *C* make a ſtock; whereof *A* has 20*l*; *B*, 30*l*. They gain 36*l*: whereof *C* got 16*l*. What was *C*'s ſtock, and the gain of *A* and *B*? (Anſw.) Take 16 from 36; and the remainder, 20, is the ſum of the gain of *A* and *B*; which, being divided in proportion to their ſtocks, gives their ſhares: Then find C's ſtock in ſuch proportion to his gain, as *A*, or *B*'s ſtock to his gain.——II. *A* put into a common ſtock 20*l*; and *B*, 144 ducats. They gaind 60*l*: of which *A* got 38*l*. What was the ducat valued at? (Anſw.) Take 38 from 60: the remainder, 22, is *B*'s gain: Then, ſay, As 38*l* (*A*'s gain) to 20*l* (his ſtock) ſo is 22*l* to a 4th term; which is *B*'s ſtock: Then, if 144 ducats give that ſtock; what is 1 ducat worth?——III. *A*, *B*, and *C*, make a common ſtock of 468*l*: with which they trade, and gain a certain ſum: whereof the ſhares of *A* and *B*, together, made 64*l*; of *B* and *C*, 58; of *A*, and *C* 70*l*. What is the particular ſtock and gain of each partner? (Anſw.) Add 64, 58, and 70: the ſum 192*l*, is double the total gain; becauſe each partner's ſhare is twice contain'd in it. Therefore the half of it, 96*l*, is the total gain: from which take 64*l* (*A* and *B*'s ſhare) the remainder, 32, is *C*'s ſhare; which, taken from 58*l* (*B* and *C*'s ſhares) leaves 26*l*, for *B*'s ſhare: which, taken from 64*l* (*A* and *B*'s ſhare) leaves 38*l*, for *A*'s ſhare: Then, having the particular gains, divide the total ſtock proportionaly.——IV. *A* has in ſtock 35*l*, and *B* 20*l*. They agreed that the gain be divided, ſo as *A* have 10 per cent, and *B* only 8. How is 40*l* to be divided betwixt them? (Anſw.) Find what is due to 35*l*, at the rate of 10 per cent; and to 20*l*, at the rate of 8 per cent: then divide the total gain (40*l*) in proportion to thoſe ſums. For, the only meaning, ſuch a queſtion can have, is, That the gain be proportion'd to what 35 would draw, of 10 per cent; and 20, of 8 per cent: and not, That *A* has realy 10 per cent; and *B*, 8; for their ſtocks. For, they will have more or leſs, according as the total gain happens to be.——V. *A* and *B* were in company, thus: *A* had 50*l*, in ſtock, for 10 months; and *B* had his ſtock in, for 8 months; and receiv'd equal ſhare of the gain: What was *B*'s ſtock? (Anſw.) Since their gain was equal, ſo muſt the products of their ſtocks, and times. Wherefore multiply *A*'s ſtock and time (to-wit, 50*l*) by 10: the product is 500; which divide by *B*'s time, 8; the quote (62*l*, 10*s*) is *B*'s ſtock: Or, which is the ſame, make this proportion: As *B*'s time (8 months) to *A*'s time (10 months) ſo, reciprocaly, is *A*'s ſtock (50*l*) to *B*'s, 62*l*, 10*s*. (Note) If we ſuppoſe *A*'s gain is to *B*'s in any other proportion, as 2 to 3: then, becauſe the gains are proportional to the products of ſtock and time; ſay, as 2 to 3: ſo is 500*l* (the product of *A*'s ſtock and time) to a 4th, to-wit, 750*l* (the product of *B*'s ſtock and time) which, therefore, divided by 8 (*B*'s time) the quote is 93*l*, 15*s*; for *B*'s ſtock.——VI. *A* receives (of gain) 20*l*, for 8 months: *B*, 25*l*; for 7 months: and *C*, 36*l*; for 5 months, The ſum of the products of their ſtocks and times is 520*l*. What were their ſtocks? (Anſw.) Divide 520*l* in 3 parts, proportiond to 20*l*, 25*l*, and 36*l*: then divide each of theſe parts by the reſpective times, 8 months, 7 months, and 5 months: the quotes are the ſtocks ſought. (Note) If inſtead of the particular times, the ſtocks were given; and the times requird: the operation is the ſame. For, 520 being reſolvd into 3 parts proportiond to the gains, divide theſe parts by the ſtocks; and the quotes are the times.——VII. *A* gains 20 *l*; and his ſtock is 15 *l* more than *B's* ſtock; whoſe gain is 12 *l*. What are the particular ſtocks? (Anſw.) Say: As the difference of the gains is to the difference of the ſtocks: ſo is each of the particular gains to the correſpondent ſtocks. For, ſince the ſum of the gains is to the ſum of the ſtocks as each gain to its ſtock: then, from the nature of proportion, the difference of gain is to the difference of ſtock, as each gain to its ſtock.——VIII. *A* gains 20 *l*, in 6 months; *B*, 18 *l*, in 5 months; and *C*, 28, in 9 months; whoſe ſtock is 72. What are the ſtocks of *A* and *B*? (Anſw.) Multiply *C*'s ſtock and time: the product is 648 *l*. Then, As 28 *l* (*C*'s gain) to 648 *l*: ſo are 20 *l*, and 18 *l*, to the products of *A*'s, and *B*'s ſtock and time; which being found; divide them by their times; and the quotes are the ſtocks. (Note) If, inſtead of the real ſums of gains, there were given 3 numbers in the ſame proportion as the real gains; the work is the ſame. (2) Or, ſuppoſe, inſtead of the particular gains, that *A* has $\frac{1}{3}$ of the whole gain; and *B*, $\frac{2}{5}$: then, we muſt add theſe fractions; and take the ſum from 1: the remainder is the fraction of the total gain, which *C* has. And, then, uſe theſe fractions, as the particular gains. (3) Again, if their particular gains and ſtocks are given, with the time of one partner; to find the times of the reſt: the work is alſo the ſame.——IX. *A*, *B*, *C* have a common ſtock of 1000 *l*. *A* gains 100 *l*, for 9 months; *B*, 80 *l*, for 12 months; and *C*, 120 *l*, for 8 months: What were the particular ſtocks? (Anſw.) Divide each partner's gain by his time; and then divide 1000 *l* into 3 parts, proportiond to thoſe quotes. (Note) If, inſtead of the total ſtock, and particular times (as above) were given the particular ſtocks, and total time: to find the particular times; the ſolution is after the ſame way, and for the ſame reaſon; to-wit, dividing the particular gains by their ſtocks; and proportioning the times to thoſe quotes. ——X. *A* has 200 *l* more ſtock than *B*: but *A* continued his, only 5 months; and *B*, 9; and drew equal gains: What are the ſtocks? (Anſw.) As the difference of times to the difference of ſtocks: ſo is *A*'s time to *B*'s ſtock; and *B*'s time to *A*'s ſtock. Or, having one ſtock; by

that, and the difference, find the other.——XI. *A*, *B* and *C* have 100*l*, to be divided among them, in ſuch manner that 2 times *A*'s ſhare be equal to 3 times *B*'s; and 4 times *B*'s be equal to 5 times *C*'s; What are their ſhares? (Anſw.) It is plain, by the conditions, that, as oft as *A* gets 3; *B* muſt have 2: alſo, as oft as *B* gets 5, ſo oft muſt *C* get 4. Then, ſay: As 5 to 4: ſo is 2 to $1\frac{3}{5}$. So that, as oft as *B* gets 2, ſo oft *C* gets $1\frac{3}{5}$: but, ſo oft, alſo, *A* gets 3: therefore the proportions of the ſhares ſought are 3, 2, $1\frac{3}{5}$; or 15, 10, 8; according to which 100*l* is to be divided. (Note) Suppoſe the conditions thus; $\frac{1}{2}$ of *A*'s ſhare is equal to $\frac{2}{3}$ of *B*'s: and $\frac{2}{3}$ of *B*'s equal to $\frac{5}{7}$ of *C*'s: we may find the proportions of their ſhares, the ſame way as before.——XII. A father, ignorant of arithmetic, orders his eſtate, of 500*l*, to be divided among 3 ſons; ſo as the eldeſt have $\frac{1}{2}$; the ſecond, $\frac{3}{5}$; and the third, $\frac{4}{7}$: What is each ſon's part? (Anſw.) Here it is impoſſible to give them theſe ſhares; becauſe $\frac{1}{2}$ $\frac{3}{5}$ and $\frac{4}{7}$ exceed the whole. And therefore the meaning of the queſtion muſt be underſtood to be, the dividing 500*l* into 3 parts, that bear ſuch proportion to one another as theſe fractions. And the like is to be underſtood of all diviſions propoſd in this manner; whether the parts propoſd exceed, or come-ſhort of, the thing to be divided. ——XIII. It is propoſd to divide 300*l* among 3 perſons, ſo *A* gets 6*l* more than $\frac{1}{2}$, *B* 12*l* more than $\frac{1}{3}$, *C* 8*l* leſs than $\frac{2}{3}$, What gets each? (Anſw.) According to the moſt obvious ſenſe of this queſtion, the meaning of it is, that the ſhares be in proportion to the ſum of 6*l*, and $\frac{1}{2}$ of 300*l*, for *A*; 12*l*, and $\frac{1}{3}$ of 300*l*, for *B*; and $\frac{2}{3}$ of 300*l* (wanting 8*l*) for *C*.——XIV. Three perſons, *A*, *B*, and *C*, buy a ſhip; of the price whereof *A* paid $\frac{5}{8}$, *B* $\frac{3}{7}$, and *C* 140*l*. How-much money paid *A*, and *B*? and What part of the ſhip had *C*? (Anſw.) Add the fractions $\frac{5}{8}$ and $\frac{3}{7}$; and take the ſum from 1: the remainder is the part of the ſhip belonging to *C*. Then, ſay, If *C*'s part coſt 140*l*; What coſt the ſum of *A* and *B*'s parts? And, having found that, divide it into 2 parts, proportiond to one another as $\frac{5}{8}$ to $\frac{3}{7}$.——XV. There were (at a feaſt) 20 men, 30 women, and 15 ſervants; for every 10*s* that a man paid, a woman paid 6, and a ſervant 2: How-much did every man, woman, and ſervant, pay of 24*l*? (Anſw.) Multiply 20 into 10, 30 into 6, and 15 into 2. Then divide 24*l* into three parts, proportiond to theſe products (to-wit, 200, 180, and 30) and you have the total paid by the 20 men, 30 women, and 15 ſervants: each of which ſums, being divided by their reſpective numbers of perſons, gives the payment made by each individual. (Note) (1) Suppoſe the conditions ſuch, that a man pays 3 times as-much as a woman, and 2*s* more: that a woman pays double of a ſervant, and 1*s* more. To find their ſhares, multiply 2 by 20, and 1 by 30: the products 40 and 30 (equal to 70*s*.) take from 24*l*: the remainder is 20*l* 10*s*. Then, becauſe a man pays triple of a woman, ſuppoſe a man pays 3, a woman pays 1; and, becauſe a woman pays double of a ſervant, if a woman pays 1, a ſervant pays ½: ſo, their proportions are 3, 1, ½; or, in whole numbers, 6, 2, 1; that is, 6, for a man; 2, for a woman; and 1, for a ſervant. Multiply theſe by their reſpective numbers of perſons: the products are 120, for 20 men; 60, for 30 women; and 15, for 15 ſervants. Then divide 20*l* 10*s*, in three parts, in proportion to one another, as 120, 60, and 15; and divide theſe parts by their reſpective numbers of men, women, and ſervants: the quotes are what each man, woman, and ſervant, pays of the 20*l* 10*s*. Laſtly, to a man's ſhare of this add 2*s*; to a woman's 1; and you have their complete payments of the whole 24*l*. (2) If, inſtead of adding, it had been propoſd to ſubtract [as if a woman pays 1*s* leſs than the double of a ſervant; then add 30*s* to 24*l* (ſubtracting what a man pays more than triple of a woman) and, in the laſt part, inſtead of adding, ſubtract 1 from the woman's part of the ſum divided.——XVI. A father, dying, left his wife with child; to whom he bequeathd (if ſhe had a ſon) ½ of his eſtate; and ⅔ to the ſon: But (if ſhe had a daughter) ⅓ to her, and ⅔ to her mother. It happend that ſhe had both a ſon, and a daughter: How ſhall the eſtate be divided, to anſwer the father's intention? (Anſw.) As the father plainly deſignd the ſon to have double of the mother's part; and the mother, double of the daughter's part: therefore, for every 1, the daughter had, the mother muſt have 2; and the ſon, 4: and in proportion to theſe numbers 1, 2, 4, muſt the eſtate be divided. (Note) Suppoſe that the mother had a ſon, and a daughter, who livd; but herſelf died in the birth: How is the eſtate to be divided betwixt the ſon, and the daughter? (Anſw.) Find the parts belonging to mother, ſon and daughter: then, divide the mother's part betwixt the children, according to the rule of heir-ſhip in the country where the queſtion ariſes.

FRACTIONS.[a]

VULGAR.

Reduction [b].

1 *TO the Loweſt térms:* each by 2'[c], or 5'[d], or other cómmon diviſor *.

2 *Tó Common-dén:* mult the déns; numer éach into áll-but-its-own den [f].

3 *To a Símple, a compound:* núm into núm; into den, den [g].

4 *Tó an Impróper, a míxt:* into den the ínteger; add num [h].

5 *To a Mixt, an improper:* númer by dénominator [i].

6 *Tó a Single, a fráction-of-fráction:* When the íntegral part is

7 (1) Núm into dén of the fráctional, plus its núm: into den, den [k].

8 (2) Dén into dén of the fráctional, plus its núm: into num, den [l].

9 *Tó a Greater name, a míxt made imprópér:* as cómpound to ſimple [m].

10 *Tó a Leſs name, a greáter:* the núm, into th' párts down to th' given [n].

11 *To a Fráction, a whóle:* 1 the dén: or, for núm; into whole, den [o].

12 *Tó Decimáls:* numerátor (with cíphers) by dénominator [p].

To

13 *Tó a certain num or dén:* as the óne to the ó-
ther, ſo ſought is [q].

* Cómmon Divíſor, *for lówering to th' leáſt*,
is the Láſt; to be found thus:
15 Gréater by léſs: the divíſor by réſt: and ſo
ón, till reſts nothing [e].

Operations.

16 [*Vúlgar*] Add [r] ánd Sub [s], the núms: having
brought 'em to óne common -nator.
17 Múltiply num into núm; into dén, den [t].
Divide, as follows:
18 Fór *numer:* num of -dénd into den of -fór:
th' others *den* give [u].
19 [*Míxt*] into th' Integer *múltiply*; by th' Frác-
tion *divíde* -cand: and *add* Both [w].

DECIMAL.

20 [*Décimals*] as I'ntegers: Tŏ A'dd, or Sub-
tract; place éach to its value [x]:
21 Cúting-off pláces in A'dd Sub, the móſt [y]: in
Múlt, both the factors [z].
22 A'nd, to the próduct, when féwer the pláces,
prefixing ciphers [aa].
23 Di'v: To the dívidend's make équal the quó-
tient's wíth the diviſor's [bb]:
24 Thérefore, when léſs; ciphers ádd to the
dívidend [cc]; prĕfíx to the quotient [dd].

Valuations

[*of Vulgar*] [*and Decimal*]

25 I'nteger (ín lower náme) into númer by dé-
nominator [ee].

Use.

Uſe.

26 USE of Décimals: ăn eaſier working than with *Vulgars* [ff].

a For the *nature* of fractions, both Vulgar and Decimal, V. Arithmetic: note [c].—As to the *operation* of fractions, it is a good obſervation of Dr. Wells's, that the common way of placing the denominator under the numerator, with a line between, occaſions ſome ſeeming difference between the operations of integers, and common fractions. In order to the taking-away of it, he adviſes to write common fractions as we write numbers of ſeveral external denominations; that is, to place the denominators at the head, or toward the top of the numerators of their reſpective common fractions. Namely, as we denote two ſhillings and ſix pence, thus, over the figures, $\overset{s.}{2}\ \overset{d.}{6}$: ſo he thinks it would be better to denote the ſame by common fractions, thus 2^{20} (to-wit, of a pound) 6^{12} (to-wit of a ſhilling) than, as uſualy, thus: $\frac{2}{20}$. and $\frac{6}{12}$. For, the former way being the ſame whereby integers of ſeveral external denominations are denoted; the working of common fractions, when expreſt after the ſame way, will more appear to be agreeable to the working of thoſe, than if the common fractions were expreſt the common way.—Thus $\frac{8}{12}$ and and $\frac{3}{12}$, with their denominators a-top, or rather affixt (as in the margin) appear to be wrought after the manner of common operations in integers.

Addit.	Subtr.	Multip.	Divis.
8^{12}	8^{12}	8^{12}	24^{12}
$+3^{12}$	-3^{12}	$\times 3^{12}$	$\div 3^{12}$
$=11^{12}$	$=5^{12}$	$=24^{144}$	$=8^{12}$

b The *reduction* of fractions being in order to prepare them for adding, ſubtracting, &c. it was neceſſary to treat of it, before an account could be given of the operations.

c *Halving* the numerators and denominators of a fraction (1) often gives a fraction in the loweſt terms: thus $\frac{20}{48}=\frac{10}{24}=\frac{5}{12}$ (2) or readily offers a common diviſor:

thus, $\frac{96}{120} = \frac{48}{60} = \frac{24}{30} = \frac{12}{15} \div 3$ (the common diviſor) $= \frac{4}{5}$.

d When the numbers end in a cipher, or 5; dividing by *five* will abbreviate a fraction. Thus $\frac{25}{350}$ will be $\frac{1}{14}$. for, $25 \div 5 = 5$: and $350 \div 5 = 70$. Then $5 \div 5 = 1$: and $70 \div 5 = 14$.

e *For example:* To bring to the loweſt term $\frac{84}{96}$; Say, 96 (the greater) $\div$ 84 (the leſſer) $= 1$: reſts, or remains 12. Then 84 (the diviſor) $\div$ 12 (the remainder) $= 7$: remains 0. Therefore 12 (the laſt diviſor) in 84, 7 times; in 96, 8 times: hence, in the loweſt terms, $\frac{7}{8}$. ——*Note*, If the laſt diviſor be 1: the fraction cannot be reduc'd to lower terms.

f *For example.* (I) $\frac{2}{3}$ and $\frac{3}{4}$: (1) for the Denom. $3 \times 4 = 12$: (2) for the Numer. $2 \times 4 = 8$, and $3 \times 3 = 9$. So, the fractions equal to $\frac{2}{3}$ and $\frac{3}{4}$ (having the ſame denom.) are $\frac{8}{12}$ and $\frac{9}{12}$. (II) $\frac{2}{3}$ and $\frac{4}{6}$ and $\frac{3}{4}$: (1) for the Denom. $3 \times 6 \times 4 = 72$ (2) for the Numer. $2 \times 6 \times 4 = 48$: and $4 \times 3 \times 4 = 48$: and $3 \times 6 \times 3 = 54$. So, the equivalent of fractions (with one common denominator) are $\frac{48}{72}$ and $\frac{48}{72}$ and $\frac{54}{72}$. V. note g.

g *Thus* $\frac{1}{2}$ of $\frac{3}{4}$ of $\frac{5}{6}$ will be $\frac{15}{48}$.——In like manner Fractions of diverſe denominations may be brought into one denomination by involving the leſs into the parts of a greater. Thus (1) $\frac{3}{4}$ of a penny may be reduc'd to the proper fraction of a greater name, to wit: $\frac{3}{4}$ of $\frac{1}{12} = \frac{3}{48}$ *s*: and $\frac{3}{4}$ of $\frac{1}{12}$ of $\frac{1}{20}$ (or $\frac{3}{4}$ of $\frac{1}{240}$) $= \frac{3}{960}$ *l*. So, (2) $\frac{3}{7}$ of an ounce, reduc'd into the fraction of a cw, will be $\frac{3}{7}$ of $\frac{1}{28}$ of $\frac{1}{4}$ of a cw $= \frac{3}{784}$. And, ſo, of any other.

h *Thus* $3\frac{2}{4} = \frac{14}{4}$: to-wit, $4 \times 3 + 2 = 14$, the numerator to 4.

i *Thus*, $\frac{14}{4} = 3\frac{2}{4}$: to-wit, $14 \div 4 = 3$; remains 2, that is $\frac{2}{4}$ or $\frac{1}{2}$.

k *For example:* To reduce $\frac{42\frac{7}{8}}{49}$ to a ſingle fraction, (1) 42 (the numerator of the integral part) $\times$ 8 (the denominator of the fractional) $+$ 7 (the numerator of the fractional) $= 343$, for the new numerator. (2) 49 (the denominator of the integral part) $\times$ 8 (the denominator of the fractional) $= 392$, for the new denominator. So, the ſingle fraction, to which this fraction-of-fraction is reduc'd, will be $\frac{343}{392} = \frac{7}{8}$.

l *For example:* To reduce $\frac{7}{19\frac{3}{5}}$ to a ſingle fraction (1) 19 (the denominator of the integral part) × 5 (the denominator of the fractional part) + 3 (the numerator of the fractional) = 98, for a new denominator. (2) 7 (the numerator of the integral part) × 5 (the denominator of the fractional) = 35, for a new numerator. So, the ſingle fraction, to which this fraction-of-fraction is reduc'd, will be $\frac{35}{98} = \frac{5}{14}$.

m *Thus,* To reduce 3 $\frac{1}{2}$*d* into the proper fraction of a pound: 3 $\frac{1}{2}$ being made an improper fraction (by line 4) to-wit, $\frac{7}{2}$: ſay $\frac{7}{2}$ of $\frac{1}{12}$ of $\frac{1}{20}$ (or $\frac{7}{2}$ of $\frac{1}{240}$) = $\frac{7}{480}$. V. note p.

n *Thus,* To reduce $\frac{7}{480}$ *l* to the fraction of a penny, of the ſame value: 7 (the numerator) × 20 (ſhillings, the next inferior denomination) = 140 × 12 (pence, the next, and ſought) = 1680. Therefore $\frac{1680}{480}$*d* = $\frac{7}{480}$ *l.*

o *Thus* 3, a whole number, expreſt fraction-wiſe is $\frac{3}{1}$. ——Or, If you would reduce a whole number to a given denominator; multiply the given denominator into the whole number: thus 3, with 6 (for a denominator) will be $\frac{18}{6}$.

p *For example:* $\frac{5}{7}$ of a pound (5.00000000 ÷ 7) will be .71428572. for, As 7 (the denom.) to 5 (the numer.) So 100000000 (the denominator, of the decimal, underſtood) to .71428572 the numerator of the decimal expreſt) *——*In like manner* may be expreſt, decimaly, the parts of Coins, Meaſures, Time, Weights, &c. E. G. (I) *Coins:* (1) 5*s* is $\frac{5}{20}$: (by line 12) = .25 : for 5.00 ÷ 20 = .25. (2) 14*s* 3*d* 2*q*. Say 14 × 12 + 3 (to reduce the *s* to *d*) = 171 *d*. Then 171 × 4 + 2 (to reduce the *d* to *q*) = 686 *q*. Then (960 being the integer reduc'd to the ſame name; to-wit, farthings) the fraction will be $\frac{686}{960}$: (V. note m) which (by line 12) will be .714; or, more fully, .71428572 †. (II) *Meaſures:* 2 pints (1 gallon, the integer) will be .25 : for 2 pints = $\frac{2}{8}$ of a gallon: which (by line 12) = .25. (III) *Time:* 45 minutes (the integer an hour) will be .75 : for $\frac{45}{60}$ (by line 12) = .75 †. (IV) *Weights:* (1) 9 *oz* (i. e. $\frac{9}{12}$) will be .75. (2) 8 *oz* 19 *dw* 8 *gr* (i. e. 4304 *gr*) will be $\frac{4304}{5760}$: reduc'd, .746 +.

* *The Reverſe*, or the reducing a decimal to a vulgar fraction——of a Denominator requird; ſay, 100000000:

71428572 : : 7 : 5.—of a Numerator requird; ſay, 71428572 : 100000000 : : 5 : 7.

† How, more readily to expreſs Mony and Time, in decimals. V. Intereſt.

+ *Thus* any one may make decimal tables, of all ſorts, for himſelf; or rather find the decimal he wants on any occaſion: which is incomparably better than to depend on tables ready made. Of theſe tables Mr Hill (p. 175) gives us 4 pages; and even requires (p. 182) one of the tables to be got-by-heart by every learner. To facilitate which, he propoſes an expedient as follows: The decimals of 1 penny (a ſhilling being the integer) are .08333; of 11 pence .91666, &c. To get them the more eaſily, read them (ſays he) thus: For 1; nought, eight, and all threes: for 11, nine, one, and all ſixes: and ſo on; as if the taking the pains to get and retain them, with the danger of loſing or miſremembering them, were preferable to dividing (for inſtance) 1.000 by 12, &c. when you have occaſion for it; and would be ſure of it: or, to multiply the integer (in lower name) into a decimal; when you want to know the value of it. E. G. $.08333 \times 12 = .99996$, almoſt 1*s*: which is the true eſtimate of the decimal. V. Intereſt——And

Thus it is (without any more ado) that *Sexageſimal fractions*, about which Mr Weſton makes ſuch a pother (arithm. p. 308—317) are reduc'd to decimal. *For example:* 365 days, 5 hours, $49'\ 16''\ 46'''$ in decimals would be 365.2425. for, $46'''$ (by line 12) is $.7666''$. Then $16'' + 46''' = 16.7666'' \div 60 = .2794'$. Then $49' + .2794' = 49.2794'$. And (minutes being the 60ths of an hour) $49.2794' \div 60 = 8213$. Then 5 h. $+ .8213$ h $= 5.8213$ h. And (hours being the 24th of a day) $5.8213 \div 24 = .2425$ days. Then 365 d $+ .2425$ d $= 365.2425$ days; the decimal ſought. ——NB. The decimals thus found, the ſolution of any queſtion, wherein there are ſexagenary progreſſionals, will be much ſhorter and eaſier. *For example:* Suppoſe the diurnal mean motion of the ſun were ſought (its annual courſe, through the whole zodiac, being perfected in 365 d. 5 h. $49'\ 16''\ 46'''$) the ſtating would be: As the annual (time aforementiond) to the diurnal time (24 hours) So is the mean annual motion (through 360 degrees) to the diurnal motion, or the number of degrees it advances in a day. The operation, in ſexageſimals, would be very laborious, requiring various reductions: but, in decimals, it will ſtand thus: 365.242555 days: 360 degrees : : 1 day: .985646374 degrees; that is (by line 20) 59′ 8″ 19‴ 37⁗ &c.

q *Thus*——In *vulgar* fractions (1) To reduce $\frac{3}{4}$ to a fraction of the ſame value, whoſe numerator ſhall be 15: Say, As 3 (the numerator) to 4 (the denominator) ſo is 15 (the numerator given) to 20, the denominator ſought, to make the fraction of the ſame value with the fraction propoſd. (2) To reduce $\frac{3}{4}$ to a fraction of the ſame value, whoſe denominator ſhall be 20: ſay (reverſely) 4 : 3 : : 20 : 15——In *decimals* the method is the ſame.

r *Thus* (I) $\frac{4}{8} + \frac{2}{8} = \frac{6}{8}$. (II) $\frac{2}{3} + \frac{4}{5}$: that is (by line 2) $\frac{10}{15} + \frac{12}{15} = \frac{22}{15}$: that is (by line 5) $1\frac{7}{15}$. (III) $\frac{7}{8}$ of a pound $+ \frac{2}{3}$ of a ſhilling $+ \frac{3}{4}$ of a penny [that is (by line 3) $\frac{7}{8}\ \frac{2}{60}\ \frac{3}{960}$: that is (by line 2) $\frac{403200}{460800}\ \frac{15360}{460800}\ \frac{1440}{460800}$] $= \frac{420000}{460800}$. (IV) $\frac{15}{42} + \frac{3}{8}$ of $\frac{2}{3}$ [that is (by line 3) $\frac{15}{42} + \frac{6}{24}$, or (by line 1) $\frac{1}{4}$; and (by line 2) $\frac{42}{168} + \frac{60}{168}$] $= \frac{102}{168}$. (V) $17\frac{3}{4} + \frac{1}{3}$ of $\frac{3}{4}$ [that is (by lines 4 and 3) $17\frac{71}{4} + \frac{6}{12}$: that is (by line 2) $\frac{352}{48} + \frac{24}{48}$] $= \frac{876}{392}$: that is (by line 5) $18\frac{12}{48}$, or $18\frac{1}{4}$. (VI) $1\frac{1}{2} + 74\frac{3}{5}$ [that is (by line 2) $1\frac{5}{10} + 74\frac{6}{10}$: and $\frac{5}{10} + \frac{6}{10} = \frac{11}{10}$: that is (by line 5) $1\frac{1}{10}$: ſo $1 + 74 + 1\frac{1}{10}$] $= 76\frac{1}{10}$.

s *Thus* (I) $\frac{1}{4} - \frac{3}{4} = \frac{2}{4}$. (II) $\frac{3}{4} - \frac{2}{3}$ [that is (by line 2) $\frac{8}{12} - \frac{9}{12}$] $= \frac{1}{12}$. (III) $\frac{2}{3}$ of $\frac{3}{4} - \frac{13}{14}$ [that is (by line 3 and 2) $\frac{224}{378} - \frac{351}{378}$] $= \frac{127}{378}$. (IV.) $\frac{3}{7}$ of $\frac{5}{9}$ of $\frac{2}{3}$ $- 7$ [that is (by line 3 and 11) $\frac{33}{189} - \frac{7}{1}$: that is (by line 2) $\frac{30}{189} - \frac{1323}{189}$] $= \frac{1293}{189}$: that is (by line 5) $6\frac{159}{189}$: that is (by line 1) $6\frac{53}{63}$. (V) $2\frac{3}{4} - 11\frac{2}{3}$ [that is (by line 4) $\frac{11}{4} - \frac{35}{3}$ (by line 2) $\frac{33}{12} - \frac{140}{12}$] $= \frac{107}{12}$: that is (by line 5) $8\frac{11}{12}$. (Or) $2\frac{3}{4} - 11\frac{2}{3}$: that is (by line 2) $2\frac{9}{12} - 11\frac{8}{12}$. Then, to ſubtract numerator 9 from 8, borrow an integer of the mixt number; and ſay, 9 from 12 (the denom.) $= 3 + 8 = 11$: then (for the integers) 1 (that I borrowd) $+ 2 = 3 - 11 = 8$: ſo the anſwer is (as afore) $8\frac{11}{12}$.

t *Thus* (I) $\frac{7}{8} \times \frac{3}{4} = \frac{21}{32}$. (II) $11\frac{2}{3} \times \frac{3}{4}$ [that is (by line 4) $\frac{35}{3} \times \frac{3}{4}$] $= \frac{105}{12}$: that is (by line 5) $8\frac{9}{12}$. (III) $11\frac{2}{3} \times 2\frac{3}{4}$ [that is (by line 4) $\frac{35}{3} \times \frac{11}{4}$] $= \frac{385}{12}$: that is (by line 5) $32\frac{11}{12}$. (IV) $\frac{2}{5} \times 7$ [that is (by line 11) $\frac{2}{5} \times \frac{7}{1}$] $= \frac{14}{5} =$ (by line 5) $2\frac{4}{5}$. (V) $7\frac{5}{8} \times 4$: that is (by line 4 and 11) $\frac{16}{8} \times \frac{4}{1} = \frac{244}{8}$: that is (by line 5) $30\frac{4}{8}$.——

Note, Multiplication of fractions diminiſhes the value, in proportion as the multiplication of whole numbers increaſes it. Thus $\frac{1}{8}l$ (or $30d$) $\times \frac{1}{8}l$ (or $30d$) $= \frac{1}{64}l = 3d$: $3q$*. So $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$. V. Leybourn, curſ. mathem. p. 38.

* The ſame in *decimals*: $.125 \times .125 = .015625 = 3d$ $3q$.——Yet it is obſervable, that the product will alter in value, according as you alter your integer. Thus, ſuppoſe the former queſtion were propounded, and a ſhilling to be the integer; then $.125l = 2.5s \times 2.5 = 6.25s$, or $6s$ $8d$.

u *Thus*, (I) $\frac{4}{9} \div \frac{3}{5} = \frac{20}{27}$. (II) $8 \div \frac{5}{6}$ [that is (by line 11) $\frac{8}{1} \div \frac{5}{6}$] $= \frac{48}{5}$. (IV) $3\frac{9}{8} \div 2$ [that is (by line 4 and 11) $\frac{33}{8} \div \frac{2}{1}$] $= \frac{43}{16}$. (V) $\frac{2}{11} \div \frac{2}{5}$ of $\frac{5}{7}$ [that is (by line 3) $\frac{2}{11} \div \frac{10}{35}$] $= \frac{70}{110}$; or (by line 1) $\frac{7}{11}$.

w *For example:*——In 276 barrels of raiſins, each $3\frac{1}{4}$ cw; How-many cw? (Anſw.) $276 \times 3 = 828$: then, $276 \div 4 = 69$: then, $828 + 69 = 897$ cw, in the 276 barrels. ——In 24 caſes of tobacco, each $2\frac{3}{4}$ cw; How-many cw? (Anſw.) $24 \times 2 = 48$: then, $24 \div \frac{2}{4}$ (or half) $= 12$; and $24 \div \frac{1}{4}$ (or quarter) $= 6$: then $48 + 12 + 6 = 66$ cw, in the 24 caſes. V. Tables: note [b].

x *That is*, Place the commas, or ſeparating points, directly under one another; and the figures, according to their reſpective values, or diſtances from unit ——And, if the numbers, given to be added, are not of the ſame denomination; they muſt be brought to the ſame. Thus (1) to *add* $.725l$ to $.625s$; having prepard the biger by multiplication (as in line 25) it will be as in the margin, at *A*; or the leſſer, by diviſion (to-wit, $62500 \div 20$) it will ſtand as at *B*. (2) To *ſubtract* $.625s$ from $725l$; the terms being prepard as afore, the work will ſtand as at *C* and *D*.

A	14.500
	625
	15.125
B	.725
	.03125
	.75625
C	14 500
	625
	13.875
D	.725
	.03125
	.69375

y *That is*, In addition and ſubtraction (having plac'd the numbers according to the directions of note [x]) add, and ſubtract them, after the manner of integers; cuting-off ſo many places as are moſt in any of the given numbers. The work will ſtand as in the margin, at *E* and *F*. *That*

E	To	3.875
	add . . .	.75
	and . . .	13.6634
	Total	18.2884
F	From	3.024
	take . . .	2.23
	Remains . .	.794

z *That is*, In multiplication, cut-off ſo many places as there are in both the factors: e. g. .75 × .5 = .375; in which product the number of places are equal to thoſe of the two factors.

aa *Thus*, .0347 × .0236 = 81892: to which [the number of places being fewer (by 3) than thoſe of both the factors] prefix 000; and the true product will be .00081892. So 36.252 × .00032 = .0116006‍4.—*Note*

I. *When any propoſd number of decimals are to be multiplied into* 10, 100, 1000, 10000, *&c.* it is only removing the ſeparating point, in the multiplicand, ſo many places towards the right-hand as there are ciphers in the multiplier. Thus, .578 × 10 = 5.78: and .578 × 100 = 57.8. Again, .578 × 1000 = 578: and .578 × 10000 = 5780.

II. *To abbreviate multiplication to ſo many places as may ſerve the purpoſe* [*].—Rule [†] (I) Place the factors in their natural order. (II) Then (having counted the number of decimals in both) from their ſum ſubtract the number you would have: and cut-off, from the right, in the multiplicand, ſo many figures as the remainder. (III) Then (1) By the 1ſt figure of the multiplier, multiply only thoſe to the left of the line of ſeparation, adding what would be carried by the multiplication of 2 figures to the right [‡]; and ſet-down the product under the firſt figure multiplied. (2) By the 2d figure of the multiplier, begin with the next figure (to the right) of the multiplicand; and ſet-down the firſt figure of the product under the firſt of the preceding multiplication. (3) By the 3d figure, take-in the next (to the right) in the multiplicand, *&c.* proceeding with the reſt in like manner——Example: 45.4789 × 6.235. (1) *At large*, the work will ſtand as in the margin at *A*. (2) *To retain only* 3 *decimals*, Take 3 from 7 (the number of all the decimals) the remainder is 4: cut-off, therefore, 4 figures of the multiplicand from the right; and begin to multiply into the next to the left, proceeding according to the above-directions; and the operation will advance as in the margin at *B*. (3) *To have only the integers*: for 7 (the ſum of the decimals) the line of ſeparation will

```
A . . 45.4 789
          6.235
       --------
       2273 945
     1 3643 67
      9 0957 8
    272 8734
           ----
    283.5609 415
```

cut off, not only all the multiplicand; but alſo 1 figure of the multiplier; as at *C*, in the Margin: and, then, the operation will proceed thus: 3 × 0 (the place to the left of 4 in the multiplicand) = 0 + 1 (that I carry from 3 times 4, the figure to the right) = 1; which I ſet-down. Then, 2 × 4 = 8 + 1 (that I carry from 2ce 5) = 9. Then, 6 × 5 = 30 + 3 (that I carry from 6 times 4, plus 6 times 7, to-wit, 28; which being above 25, I carry 3) = 33. Then 6 × 4 = 24 + 3 (that I carry) = 27 †‡.

$$B \;\ldots\ldots\ldots\ldots\ldots\ldots \quad \begin{array}{r} 45|.4\,789 \\ \underline{6.235} \end{array}$$

$$\left.\begin{array}{l} 45\cdots \\ 454\cdot\cdot \\ 4547\cdot \\ 45478 \end{array}\right\} \times \left\{\begin{array}{l} 5\cdots\cdots\; 227 \\ 3\cdots\cdot 1\,364 \\ 2\cdots\cdot 9\,096 \\ 6\cdot\cdot 272\,873 \end{array}\right.$$

$$\overline{\;283.560\;}$$

$$C\left\{\begin{array}{r} 45.4\,78\,9 \\ \underline{6.23|5} \\ 1 \\ 9 \\ \underline{273} \\ 283 \end{array}\right.$$

* For the moſt part we have occaſion for *no more than* three or four figures after the ſeparatrix. In ſuch caſes, to cut-off the following places will be of great eaſe and uſe, on many occaſions: more particularly in reſolving adfected equations; in calculating of trigonometrical problems by the natural ſines and tangents, &c.

† *The common Method,* in this caſe, that has been handed-down from Dr. Record (1327) to Mr. Holiday (1746) is to reverſe the order of the figures of the multiplicator (which is unnatural, and troubleſome) and to have a careful regard to ſet-down the unit's place, in the multiplier, underneath the proper figure of the multiplicand, according to the deſignd product; which adds (not a little) to the encumbrance.——That, which I have here given (how-obvious-ſoever it may appear, becauſe natural and eaſy) was firſt publiſht by Capt. Wilkinſon, in his beautiful tables of intereſt, &c. to whom, therefore, the curious are obligd for the uſeful diſcovery.

‡ *As to the Allowance for what may be carried from the columns neglected,* Mr Malcolm (p. 142) ſays it is altogether a gueſs; inaſmuch as we may, very often, make the product leſs than it ought to be, by 1 or 2, in the laſt place; which, he adds, can ſcarcely be helpt, otherwiſe than by making one or two more columns than the num-

ber of decimal places one would have in the product; and ſo cutting-off the two laſt places from the product. ——How this may be in the common method, I have not conſiderd: but, for ought I have found in a variety of caſes in Capt. Wilkinſon's method, there is a ſufficient certainty, by obſerving the following *Rule*: to-wit,

If the next figure on the right hand of that you begin-with (in the multiplicand) multiplied into the figure of the multiplicator you are working-with, gives a product betwixt 5 and 15, carry 1: if the product be above 15 and leſs than 25, carry 2: and, if it ariſe to any number betwixt 25 and 35, carry 3: and, ſo, on.

†† Note—*In caſe of decimals only*, the procedure is the ſame. Thus, .23456 × .23456. (1) To have a product of 6 figures, the work will ſtand as at *D*. (2) To have a product of 3 figures, the line of ſeparation being plac'd ſo as to cut-off two figures of the multiplier (according to the preceding directions) the operation will come-out, as at *E*—*NB* (1) To the products of both (*D* and *E*) is prefixt a cipher; to ſupply the defect of one place, according to the inſtructions given, line 22. (2) To prevent miſtakes of the eye, in the operation; it may be convenient to dot the figures, as you proceed, both in the multiplicator and in the multiplicand.——This method of contraction may ſometimes be uſd with advantage *in whole numbers*: as in the following aſtronomical calculation, and the like. Let the ſun's declination be requird, when it is in taurus 27°. 34′; its greateſt declination being 23° 29′. This will be found by the rule of proportion, thus: As the radius to the ſine of the ſun's greateſt declination: ſo is the ſine of its longitude, to the ſine of its preſent declination, &c. that

```
D  . . .2|3456
          .2 3456
         ---------
       14
      117
      938
     7037
    46912
    -----
  .055018
```

```
E †  . . 1
         7
        47
       ---
      .055
```

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† Say, *as at* C (p. 76) 4 × 0 (*the place to the left of 2 in the multiplicand*) = 0 + 1 (*that I carry from 4 times 2, to the right*) = 1. *Then* 3 × 2 = 6 + 1 (*that I carry from 3 times 3*) = 7: &c.

```
            Full way                       Abridg'd
F . . 100000 : 39848 : : 84402      G . . |39848
               84402                       84402
              |79696                           1
          159 |3920                          159
         1593 |92                           1594
        31878 |4                           31878
10000) 33632 |50896 (19° 39'               33632
```

that is as in the margin at *F*: where obſerve, that, becauſe the five laſt figures of the product are to be cut-off, for the five ciphers of the diviſor; the remaining figures (to-wit, the five firſt) being the quotient, and ſufficient for this uſe, may be ſecurd by abridging the work; as at *G*.

bb *Thus*, (I) .8030 ÷ .22 — 3.65 (II) 8.030 ÷ 22 = .365. (III) 8321.9 ÷ 73.2 = 113.

cc *Thus* (I) 22 ÷ .365 [that is (ciphers added to the dividend) 22.000 ÷ .365] = 60: Or, 22.0000 ÷ .365 = 60.2. (II) 22 ÷ 3.65 [to-wit, 22.0000 ÷ 3.65] = 6.02.

dd *Thus* 7.25406 ÷ 957 = 758: to which (to make-up the places of parts requird) 2 ciphers muſt be prefixt: ſo the quotient will be .00758.——*Note*

I. *When any propoſd number of decimals are to be divided by an unit, with any number of ciphers after it*; it is only removing the ſeparatrix ſo many places towards the left as there are ciphers in the diviſor. For example: If 17.28 were to be divided by 10 the quotient will be 1.728: by 100 = .1728: by 1000 = .01728: by 10000 = .001728.

II. *To Abbreviate diviſion to ſo many places as may ſerve the purpoſe*, Mr. Ward (p. 68) gives us an expedient: but as the attendances in the execution of it are not a little encumbering; and the way of dividing, which I have propoſd will bring the operation into leſs compaſs than his contracted form: it would be a fault to give a detail of it here.

III. *The Remainder*, after diviſion, is neglected in the application of decimals: though it is certain that, where there is a remainder, the diviſion is not perfect——The deficiency, in this caſe, depending on the true value of the remainder, Mr Malcolm (p. 144) has ſhown us, as a part of the theory fit to be known, firſt how that deficiency is to be found; and then what is to be added to the quote already found, to make the complete quote.

ee *For example:*——The $\frac{5}{7}$ of a pound, What? Anſw. (I) 5 (the numer.) × 20 (*s.* to-wit, 1 *l:* or the integer in the next lower denomination) = 100 ÷ 7 (the denom.) 14*s.* remains 2; to-wit, $\frac{2}{7}$ of a ſhilling. (II) Then (to value that) 2 × 12 (*d.* to-wit, 1 *s*; the next lower denomination) = 24 ÷ 7 = 3: remains 3; to-wit, $\frac{3}{7}$ of a penny. (III) 3 × 4 (*q.* to-wit, 1 *d*) = 12 ÷ 7 = 1 *q.* and $\frac{5}{7}$ of a farthing; *i. e.* almoſt another.——.714 of a pound, What? Anſw. Multiply the decimals, as in the margin; the value will appear to be nearly the ſame as that of a $\frac{5}{7}$. V. Tables note [b].

Decimal	*Vulgar*		
.714	5	×	20
20	7	÷	100 = 14*s.*
s. 14.280	remains		2
12	2	×	12
d. 3.360	7	÷	24 = 3*d.*
4	remains		3
q. 1.440	3	×	4
and .$\frac{440}{1000}$	7	÷	12 = 1*q.*
or . . . $\frac{11}{25}$	remains		5 = $\frac{5}{7}$

ff *The great Advantage,* propoſd by the uſe of decimal fractions, is a more ſimple and eaſy operation than what vulgar fractions (taken, either in their proper form, or as mixt integers) do require. How the application is made, for anſwering that end, and how far it is a real advantage; will appear from the following conſiderations. (1) In the firſt place, this is very evident, that if, inſtead of the ſubdiviſion of coins, weights, and meaſures, and other kinds of quantities uſeful in ſociety, which now obtain; there were one ſtandard ſuperior ſpecies, and all the ſubdiviſions were decimals (whether the ſeveral parts were alſo diſtinguiſht by names, or only by their decimal denominations: it were the ſame thing to the purpoſe) then the common operations would be as ſimple and eaſy as whole numbers: the rules and reaſons of which may appear from the preceding doctrin. But, ſuppoſing this were ſo, yet either we could not entirely avoid the conſideration of vulgar fractions, or we muſt admit of ſome inaccuracies in calculations, which are unavoidable with decimals; and which will be of more or leſs conſequence, in different circumſtances. For, we have ſeen that decimals will have remainders (becauſe every number is not an aliquot part of every other) and then the quote is not complete without bringing-in a vulgar fraction: and therefore, if we take the quote without this correction, it is leſs than juſt, according to the value of the remainder, or rather the value of the vulgar fraction that is neceſſary to complete it. Now, if the number, found by this diviſion, is the final anſwer of a queſtion, which is to be applied in no farther calculation, than if it is brought ſo low as to be leſs than any quantity of that kind that is uſd (for example the ſmalleſt real coin, or weight, *&c.* that has any name, or diſtinct being in ſociety) then the defect is not to be complaind-of; becauſe, if you do complete the quote, the additional part is of no uſe. But, if a quote is to be further employ'd in calculation, eſpecially if it is to be multiplied; the defect may become conſiderable; and it will be the more ſo, as the multiplier is greater, and alſo according to the value of the integer.—Now the only remedy for this; while we uſe none but decimal fractions, is to bring the diviſion very low; that is, to carry-it-on till the denominator be very large and conſequently what is deficient be very little. But, then, this inconveniency will frequently happen; that, by this means, we ſhall have large numbers to work-with, which will prove more troubleſome than the method of vulgar fractions. (2) The uſe and application of decimals will more fully appear under the head of intereſt, annuities, &c. As to which application this in general only needs to be further obſervd here, that, any integer being conſiderd as the higheſt denomination, all numbers, or quantities, leſs than that, are to be expreſt, decimaly, by taking the decimal of that integer, anſwering to that leſſer quantity; and, in the ſame queſtion, uſing decimals of the ſame integer for all numbers of the ſame kind (that is, for all numbers of mony uſe the ſame integer as 1 *l.*) Then multiply, and divide, by theſe numbers, according to the rules of decimals. (3) In the laſt place, it may be obſerv'd that moſt queſtions, in common buſineſs, are ſooner done without decimals, by the common method of reduction. But, when to uſe decimals, or the common methods; muſt be left to every body's own choice: and, indeed, a good deal of practiſe will be neceſſary to enable one to chooſe judiciouſly.

GAIN *and* LOSS.

1 G'AIN find the príce: then As ág to gain-ág; ſo the príce to the anſwer [a].

2 Lóſs take from ág: then As ág to the loſs; ſo the price to the anſwer [b].

a *For example:* A draper buys 2795 ells flemiſh of ghenting at 22 $\frac{1}{2}$ *d* the ell engliſh. It is requird to know at what price the cloth muſt be ſold, to gain 15 *l* 10 *s* per cent.——Anſw. The price, found by the rule of three, is *l* 157 4 4 1. Then, As 100 *l* to *l* 115 10 (that is, the gain + 100) ſo *l* 157 4 4 1 (the price given for the cloth) to *l* 181 11 9 $\frac{1}{60}$ (the price, for which it is to be ſold, to gain 15 *l* 10 *s* per cent.)

b *For example:* Suppoſe the aforeſaid cloth were to be ſold, ſo as to loſe 15 *l* 10 *s* per cent.——(Anſw.) Firſt, the loſs being ſubtracted from 100 *l*, leaves 84 *l* 10 *s*. Then, As 100 to 84 *l* 10 *s* (the value of 100, the loſs being deducted) So *l* 157 4 4 1 (the price given for the cloth) to *l* 132 16 11 3 $\frac{102}{120}$ (the price it muſt be ſold at, to loſe at the rate of 15 *l* 10 *s*. per cent.)

To theſe may be added a ſpecimen of *Such queſtions, as require ſome caſting-about to prepare them for a ſolution.* ——I. Having ſold 2 yards of cloth for 11 *s* 6 *d*, I gain'd at the rate of 15 per cent; but, had I ſold it for 12 *s*, What is the rate of the gain per cent? (Anſw.) (1) Say as 11 *s* 6 *d* is in proportion to 115 *l*: ſo is 12 *s* to a 4th term; which I find to be 120 *l*: and ſo 20 *l* is the anſwer of the queſtion. (2) Or, from the 1ſt ſale and rate of gain, find the prime coſt of 1 yard. It is 10 *s* (for, as 15 to 100; ſo is 11 *s* to 10 *s*) Then, the 2d ſale being 12 *s*, the gain is 2 *s*. Therefore, ſay, If 10 gain 2, what will 100 gain? It is 20 ——II. Having bought a parcel of goods for 18 *l*, and ſold the ſame immediately for 25 *l*, with 4 months credit: What is gaind per cent per annum? . . . Say, by the rule of five, If 18 *l* in 4 months gain 7; what will 100 *l* gain in 1 year? (1) If the gain per cent per annum is given (ſuppoſe 12 per cent) to find the time that ought to be allowd, ſay, If 100 *l* gain

12*l* in one year, In what time muſt 18*l* gain 7*l*? (2) Or, if the rate of gain is given, with the prime coſt and time, To find the ſelling price, ſay, If 100*l* in 1 year gain 12*l*, What muſt 18*l* gain in 4 months? which is to be added to the prime coſt. (3) If the rate of gain, time, and ſelling-price are given, To find the prime coſt (as ſuppoſe 4 months allow'd for payment of 25*l*, by what was gain'd at the rate of 12*l* per cent per annum) Firſt find 4 months intereſt of 100*l* at the propoſd rate, which add to 100: Then ſay, As that ſum is to 100, ſo is 25*l* to a 4th term; which is the ſum ſought.

GOLDEN RULE.

1 GÓLDEN [a]: one ſtáting all quéſtions re-
ſólves in propórtion [b]; the Rule this:
2 *Conditional*, in one line; and, óppoſite, thĕ
térms *Correſponding* [c].
3 -D'END is the *-ducing* of óne, into *-duc'd* of
the óther; the reſt -SOR [d].
4 *Nó -duc'd:* the fácit of óne line divide by
thát of the other [e].

a *So*, may this rule be well calld, by way of eminence; though the name is commonly appropriated to what is uſualy calld the rule-of-three. V. Rule-of-three: under which head is handled (moſt commodiouſly) the doctrin of proportion, as generaly deliverd by arithmeticians.

b *Whether* Single [1], or Compound [2]; Direct †, or Inverſe ⸸.—[1] *Single*, when 3 numbers are given, to find a 4th proportional. † Direct, when more requires more; or leſs, leſs. V. Note [e]. examp. I. ⸸ Inverſe, or Reciprocal, when more requires leſs; or leſs, more. V. examp. II.— [2] *Compound*, when 5 numbers are given, to find a 6th proportional. † Direct. V. examp. III. ⸸ Inverſe. V. examp. IV.

c *For examples* of placing the terms, See the indenture of note [e].

d *That is,* The producing [1] terms of one line, multiplied into the produced [2] of the other, give the dividend: and the reſt of the terms, multiplied-together, give the diviſor. So the quotient (or anſwer) falls to the blank. —[1] *Producing* terms are ſuch as jointly produce an effect; or, that are conſiderd as a cauſe, with the adjuncts of time, meaſure, diſtance, *&c.*—[2] *Produced* terms are ſuch as are connected with the others under the character of price, purchaſe, produce, gain, loſs, intereſt, advantage, value, or quantity of work, *&c.*

EXAMPLES.

Single ..... *Compound*

(Rule of 3)

I { 2 *lb* 4 *s*
6 }

II { 4 qrs ... *Broad*
5 10 *Long* }

III { *l* 6, intereſt : *l* 100, principal : 12 mo.
200 18 }

IV { 4 horſes : 8 days : 6 buſh.
16 21 }

e *To exemplify* the whole —*Queſt.* (I) If 2 *lb* of tobacco coſt 4 *s*: what will 6? (II) How--much ſtuff (yard-broad) will line 10 yards of cloth, yard-and-quarter broad? (III) At the rate of 6 per cent, per annum; What is the intereſt of 200 *l* for 18 months? (IV) If 6 buſhels of oats will ſerve 4 horſes 8 days; How-many days will 21 buſhels ſerve 16 horſes?—*Anſw.* (I) 4*s* (the price, or the produced term of one line) × 6 (the producing term of the other) = 24 (the dividend) ÷ 2 (the reſt, or remaining term) = 12 (for the blank, to-wit) the anſwer. (II) 5 × 10 = 50 (the facit of one line, for the dividend; there being no produced term) ÷ 4 (of the other line) = $12\frac{2}{4}$ or $\frac{1}{2}$ yards. (III) 6 × 18 × 200 = 21600 ÷ (100 × 12) 1200 = 18 *l.* (IV) 21 × 4 × 8 = 672 ÷ (6 × 16) 96 = 7 days.—*Note:* This anſwerd by two poſitions in the Double rule-of-three * (1) The former is in direct proportion: 6 buſhels : 8 days : : 21 buſhels : 28 days. (2) The latter, in reciprocal proportion: 4 horſes: 28 days : : 16 horſes : 7 days.

* *So* this rule is calld by ſome, on account of its having two ſtatings, which (in ſome problems) are requird for the anſwer: by others, the *rule-of-five*; by reaſon

of its having 5 numbers given, to find a 6th.—*NB.* All queſtions of this rule are ſuch as include two queſtions of the rule-of-three, ſo dependent upon one-another, that the anſwer of the firſt being made the middle term of the ſecond, the anſwers of both have the ſame ſignification; and the laſt is the final anſwer of the queſtion.

INTEREST.

SIMPLE

1 INTEREST [a] ínto a húndredth o'th' Rate,
the Príncipal and Time [b].
2 (I) '*Amount* to Fínd: add the príncipal tŏ
th' íntereſt thence accruing [c].
3 (II) *Príncipal* tŏ Fínd: The amount, by
tíme-into-rate, more unit [d].
4 (III) *Ráte*; int by tíme-into-príncipal [e];
(IV) *Tíme*; int by ráte-into-príncipal [f].

COMPOUND

5 I'NT. *upon* I'NT. (V) Into th' húndredth o'th'
Ráte, the *Amóunt* of each year gives [g].
6 Or, into P′, power of Ráte; to-wit, ínto it
ſelf, leſs 1 time [h].
7 (VI) *Príncipal:* A′ by R [i]. (VII) *Tíme:* A
by P′, Rate invólve to the quotient [k].
8 (VIII) *Rate:* Amount by Príncipal: Quote
evólve to the root-power [l].

READILY TO EXPRESS
the decimal fractions of Money:

Double

9 Dóuble fírſt ĭs Shíllings: i'th' ſécond 5''
O'ne: reſt and 3d are Farthings.
10 Báting 1 'bove 23: adding hálf for each 5'
in the 4th place [m].

Time.

11 Half-year ,ú: Quarter ,el: Month ,zeitit:
Dáy ,yydoio [n].

READILY TO FIND
the number of days

from ſuch a day in one month to ſuch a day in another.

12 Dáys to the End of the fírſt, from begínning
o'th' láſt, add-together:
13 Months-between, into ib: leſs ónes for
months íz: and for feb, three [o].

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a *Intereſt* is the uſe, or ſum of mony, reckond for the loan or forbearance of ſome *principal* ſum, lent for, or due at, ſome certain *time*; according to ſome certain *rate* (per cent. that is) by the hundred. (I) SIMPLE intereſt, according to law, ought not to be above 5 pound for the uſe of 100, for 1 year; and 10 for the uſe of 100, for 2 years; and ſo on, for a greater, or leſſer ſum in proportion to the time propoſd. (II) COMPOUND is that, which is counted from the principal, and ſimple intereſt forborn; as the intereſt ſtill becomes due: thence calld intereſt upon intereſt. This, being renderd illegal *, is ſeldom allowd; except it be by particular contract, or in purchaſing of annuities, or taking leaſes in reverſion, &c. V. Annuities.——NB. *The ſhort way* of expreſſing the problems in the doctrin of intereſt and its appendants (which I have given in brackets, to catch the eye) (I) may be preſented by the initial, or commanding letters: to-wit, *a*, amount; *i*, intereſt; *n*, annuity; *ſ*, ſum; *p*, principal; *r*, rate; *t*, time; *w*, preſent worth: (II) and, in the diſpoſition of them, it may be obſervd that letters (1) put-together, like a word, denote Multiplication; (2) on each ſide of a line, Diviſion; (3) when ſuperior, Involution. . . Thus (1) $prt = i$ (note [b]) imports that the Principal, Rate, and Time, multiplied into each other, give the Intereſt. (2) $\frac{a}{rt} + 1 = p$ (note [d]) that the Amount (divided by the product of Rate, multiplied into theTime) plus 1, gives the Principal. (3) $\frac{a}{p} = r^t$ (note [k]) that the Amount divided by the Principal, gives the Rate involvd into the Time, *&c.*

* But, abſtracting from the reaſon of the law (which may be the encouraging of trade, by employing mony, that way, rather than upon intereſt) If taking intereſt be (at-all) juſt; compound intereſt cannot be unreaſonable. For, if I can demand my intereſt, when it is due; I may take that intereſt and lend it out again upon intereſt to any other perſon. Why, then, may I not lend it out alſo to the perſon who has my principal ſum? And, in point of right and juſtice, it is the ſame thing if I continue, or have that intereſt in his hands. There is the ſame reaſon that it ſhould bear intereſt, after it becomes due; as that the original ſum ſhould do ſo.

b *For example:* [Given prt; to find I] What is the intereſt of 200*l* at 5 per cent (1) for a year; (2) for half a year (3) for 91 days.——Anſw. [$prt = i$] (1) 200 (the principal) × ,05 ($\frac{1}{100}$dth o'th' rate) = 10,00 (that is, 10*l* intereſt for 200*l*) for a year. (2) 10,00 (the intereſt of 200*l* for 1 year) × ,5 (the $\frac{5}{10}$ or $\frac{1}{2}$ of a year) = 5,000 (that is 5*l* for half a year. (3) 10,00 (the principal into $\frac{1}{100}$th of the rate, as before) × .24934 (the time. V. lin. 11.) = 2,4934000 (that is £ 2 9 10 2) for 91 days *.———*NB.* (1) This rule may ſerve for an intereſt-table in the Head, quite eaſy to uſe, and always ready at hand; and therefore preferable to any in Books, be they ever ſo correct (which yet is a hazard) ſince in the conſulting of them (beſide the inconvenience of turning to them, and the trouble of finding the particulars, and the danger of miſtakes in copying them) there can be no great ſatisfaction in the uſe of them; as it is taking things upon truſt, with an implicit faith †.

* By the rule of proportion, the laſt example would ſtand

thus: If 100*l* (principal) in 365 days, give 5*l* (intereſt) What will 200*l* (principal) give in 91 days? (Anſw.) According to the inſtructions given under the golden rule, the queſtion will be as in the margin. Then 5 × 200 × 91 = 91000: and 100 × 365 = 36500. Then 91000 ÷ 36500 = £ 2 9 10 1 $\frac{15000}{36500}$.—*NB.* By the directions given under this head of intereſt, may be known the *proviſion*, *commiſſion*, or *factorage* of any ſum: as alſo *inſurance*, *average*, *ſtorage*, *brokerage*; and *any thing elſe*, *rated at ſo much per cent.*

Princ.	Days	Inter.
100	365	5
200	91	

† Thoſe, therefore, who are employd in ſuch calculations, ought to underſtand the rules at large; and ſo be able to examin, and make tables for themſelves. And, in my opinion, it is not fit, in queſtions of conſequence, to truſt to any tables; but what one has examind, or made for himſelf. Malcolm, arithm. 6. 10. p. 614.

c *For example:* [Given p t r to find A] What will 256*l* 10*s* *amount* to in 3 years, 1 quarter, 2 months, and 18 days; at 6 per cent, per annum?—Anſw. [p t r + p = A] 3.46599 (the time) × .06 (the rate) = .2079594 × 256.5 (the principal) = 53.341586 (the intereſt) + 256.5 (the principal) = 309.841586 (the amount) that is £ 309 16 10.

d *For example:*—[Given a r t (or i r t) to find P] What *principal*, being put to intereſt, will raiſe a ſtock of £ 309 16 10; in 3 years, 1 quarter, 2 months, and 18 days; at 6 per cent, per annum? Or, What is that ſum (ſo much time hence) worth, in ready money; abating, or diſcounting 6 per cent?—(I) $\left[\frac{a}{r\,t} + 1 = P\right]$ (the rate into time; (to-wit) 3.46599 × 06) + 1 (or unit) = 1.2079504. Then 309,841586 (the amount) ÷ 1.2079594 (the time into rate + 1) = 256.5 (that is £ 256 10) for the principal ſought. (II) $\left[\frac{i}{r\,t} = P\right]$ 53.341586 (the intereſt) ÷ .2079594 (the product of time into rate) = 256.5 (the principal).

e *For example:* [Given a p t, to find R] At what *rate* of intereſt, per cent, will £ 256 10 amount to £ 309 16 10, in 3 years, 1 quarter, 2 months, and 18 days? —Anſw. [a—p ÷ t p = R] 309.841586 (the amount)

—256.5

—256.5 (the principal) = 53.341586 (the amount, leſs the principal; that is, the intereſt) ÷ 889.026435 (time into principal, to-wit, 3.46599 × 256.5) = .06, the rate requird.

f *For example:* [Given a p r, to find T] In what *time* will £ 256 10 raiſe a ſtock of (or, amount-to) £ 309 16 10, at 6 per cent?—Anſw. $\left[\frac{a-p}{rt}=T\right]$ 309.841586 (the amount) — 256.5 (the principal) = 53.341586 (amount, leſs the principal; that is, the intereſt) ÷ 15.39 (rate into principal, to-wit, 256.5 × .06) = 3.46599 (the time requird) to-wit, 3 year, 1 quarter, 2 months, and 18 days. V. lin. 11.

g *For example:* [Given p t r, to find A] £ 275 11 3, forborn 4 years, at 6 per cent, per annum, *intereſt upon intereſt:* What will it amount-to?—Anſw. as follows;

The Principal . . . . . . 275.5625 . . . × .06 $\frac{1}{100}$ o'th' Rate
gives, for Intereſt . . . . 16.5337* . . . . which + to the Principal
amounts (for the I year) to 292.0962 . . . . which × into the Rate
gives, for Intereſt . . . . . 17.5257 . . . . which + to the Principal
amounts (for the II year) to 309.6219 . . . . And, in like manner,

proceeding on, the amount (in IV years) will be found 347.8911; that is £ 347 17 10.

* *To abbreviate* the work, the two firſt places (to the right) are omited in multiplying the principal by .06; 4, or 5 places of decimals, being correct to a farthing, or little more.

h *For example:* In anſwer to the preceding queſtion [$p \times r^t = A$] 275.5625 (the principal) × 1.26247696 (the power of the rate*) = 347.8903 † that is (nearly) £ 147 17 10.

* *By the Rate* is underſtood the amount of 1 *l*, and 1 year's ſimple intereſt. So, intereſt being at 5 *l* per cent, the rate (or amount of it) is 1.05 *l:* for, 100 : 105 :: 1 : 1.05.—*The Power of the Rate* is the amount of 1 *l* for the time and rate given; and *is found* by multiplying it into it-ſelf, till the number of multiplications be equal to the time, leſs 1.

† *NB.* If the rate of intereſt is determind to any other time than a year, as $\frac{1}{2}$, or $\frac{1}{4}$; the rule is the ſame: only taking, for $\frac{1}{2}$ a year, continualy divided by the rate, till the quotient be a cipher; the number of diviſions will be the time. V. Evolution, and Note k.

i *For example:* [Given a r t, to find P] What principal ſum will amount to 201.9963136*l*, in 4 years, at the rate of 6 per cent, compound intereſt?——Anſw. $\left[\frac{a}{r}=P\right]$ 1.06 (the rate) × into it ſelf 4 times leſs one (for the biquadrate root) = 1.26247696. Then 201.9963136 ÷ 1.26247696 = 160 (the principal ſought).

k *For example:* [Given a p r, to find T] At 5 per cent, compound intereſt, in what time will 50 *l* amount to 60.7753125*l*?——Anſw. $\left[\frac{a}{p}=r^t\right]$ 60.7753125 (the amount) ÷ 50 (the principal) = 1.21550625: which is $\overline{1.05}^{4}$, or 1.05 × 1.05 × 1.05 × 1.05: which, therefore, divided by 1.05, till the quotient be a cipher; the number of the diviſions will give the time, to-wit, 4 years.

l *For example:* [Given a p t, to find R] At what rate of compound intereſt will 50 *l* amount to 60.7753125 in 4 years?——Anſw. $\left[\frac{a}{p}=r^t\right]$ 60.7753125 (the amount) ÷ 50 (the principal) = 1.21550625: whoſe 4th root (to be found by evolution, or the table of the powers) is 1.05, the rate; or amount of 1 *l* for 1 year.

Note. As the amount of any principal is the ſum of the principal and intereſt: ſo, if (in any of the preceding problems) the intereſt is ſought, or given, inſtead of the amount; the anſwer is eaſily found from the preceding. ——For example: (1) Having the principal, rate, and time; to find the intereſt . . Rule: Find the amount (by problem V) The difference of this, and the principal, is the intereſt. (2) Having the intereſt, time, and rate; to find the principal . . Rule. Find the amount of 1 *l* for the given time and rate: then, the difference of 1 *l* and that amount, being the intereſt of 1 *l*; ſay: As that intereſt is to 1 *l*: So is the given intereſt to its principal ſought. (3) Having the principal, intereſt, and rate; to find the time . . Rule: The ſum of principal and intereſt is the amount; by which, with the rate and principal, find the time by Problem VII.

m *For example:* 6.7777 = £ 6 15 6 2. for (1) The *firſt* doubled is 14 *s*: and, (2) in the *ſecond* place, 5 out of 7 is 1 *s*; in all 15 .. Then (3) the *reſt* of the ſecond figure 7 (to-wit 2) prefixt to the *third* is 27, *i. e.* (bating 1, above 23) 26 *q*, to-wit, 6 *d* 2 *q*.—NB. (I) The Reaſon of this rule is thus: (1) Since 1 ſhilling is the $\frac{1}{20}$ of a pound; and double any number of 10th parts, makes ſo many 20th parts; (ſo $\frac{1}{10} = \frac{2}{20}$) therefore double the figure in the firſt place (whoſe denominator is 10th parts) is equal to ſo many 20th parts, or ſhillings. Again, (2) Since $\frac{1}{20} = \frac{5}{100}$, therefore 5, in the 2d place (whoſe denominator is 100th parts) is 1 ſhilling. Then (3) the figure in the 3d place has 1000th parts for its denominator: and this, with the number over 5 in the 2d place, makes ſo many 1000th parts; which is little leſs than ſo many farthings: becauſe 1 farthing is $\frac{1}{960}$ part of a pound. But, when we make-up a decimal table for farthings from 1 to 47 (which is 11*d* 3*f*) we find this true in fact; that, from 1 to 23 farthings, the figures in the 2d and 3d places of the decimal are the ſame with the number of farthings: But, from 24 to 47, the figures in the 2d and 3d places make a number one more than the number of farthings. And, though in all theſe decimals (except that for 6*d* or 24*f*) there are figures after the 3d place; yet their value is not 1 farthing, becauſe they do not make ,001, which is leſs than 1 farthing. (II) Theſe things well conſiderd, the learner will find that he has little or no occaſion for ſuch large tables as are uſualy given in books of decimal arithmetic. V. Fractions.

n Accordingly, to expreſs (decimaly) the time ſpecified in the margin; the ſeveral denominations, added together, give the integer with the decimal fractions.

3 years	3		
1 quarter .	,25	: el.	zeitit
2 months .	,16667	to-wit,	2 × ,08333
18 days . . .	,04932	to-wit,	18 × ,00274
	———		———
	3,46599		yydoio

o *For example:* Suppoſe I am to receive intereſt on march 26, 1738, for mony which I lent june 21, 1725 ——What is the time it has been out?——(Anſw.) 1725 — 1737 = 12 years (from june 21, 1725; to june 21, 1737) Then, for the days from june 21 (1737) to march 26 (1738) Say: 9 (the number of days from june 21 to the end) + 25 (the number of days from the beginning of march following to the 26th) = 34. (3) Then, 8 (the number of months between june and march) × 31

= 248

= 248 — 2 (for ſeptember and november, months of 30 days) + 3 (for february) = 243 (days) + 34 (the days in june and march) = 277; in all 12 years, 277 days.

MEASURING.

1 MEA'SURE *Square:* léngth into breádth: whether by' the Foot, Yard, Square, or Rod [A].

2 Meáſure *Cube:* léngth into breádth into dépth: whether by Feet, or Yard [B].

3 *Wrought* . . by Décimals . . by A'liquots . . ór by Croſs-múltiplication [C].

A *That is,* The Content of work, taken in feet, inches, &c. is given——by the *Foot-ſquare,* in Glazing [a], and Maſonry [b],——by the *Yard-ſquare,* in Joinery [c], Painting [d], Plaſtering [e]——by the *Square-of-ten-feet,* in Flooring [f], Partitioning [g], Roofing [h], Slating and Tiling [i]—— by the *Rod-ſquare,* in Brick-work [k].

a How-many Feet of *glazing* in a pane of glaſs, which is 5 feet 73 by 2 feet 54?——Anſwer (by Decimals) 5.73 × 2.54 = 14.5542, i. e. 14 *f* 7.0504 *i.*

b How-many Feet of *paving* in a yard, which is 22 *f* 4 *i* long, and 19 *f* 7 *i* broad?——Anſwer (by Croſs-multiplication) 22 4 × 19 7 = 437 *f* 4 *i* 4 *p.*

c How-many Yards of *wainſcot* in a room, whoſe height is 12 *f* 3 *i,* and compaſs 104 *f* 6 *i*?——Anſw. 104 6 × 12 3 = 1280 1 6 ÷ 9 (the ~~inche~~s feet in a yard) = 142 *y* 2 *f* 1 *i* 6 *p.*

d How-many Yards of *painting* in a room, which is 12 *f* 4 *i* high, and 84 *f* 11 *i* about?——Anſw. 84 11 × 12 4 = 1047 3 8 ÷ 9 = 116 *y* 3 *f* 3 *i* 8 *p.*

e How-many Yards of *plaſtering* in a cieling, which is 47 *f* 4 *i* 8 *p* long, and 18 *f* broad?——Anſw. 47 4 8 × 18 = 852 10 6 ÷ 9 = 94 *y* 6 *f* 10 *i* 6 *p.*

f How-many Squares in a *floor* 49 *f* 7 *i* 4 *p* long, and 26 *f* 6 *i* broad?——Anſw. 49 7 4 × 26 6 = 1314 8 4 ÷ 10 = 13 *ſ* 14 *f* 8 *i* 4 *p.*

How-g How-many Squares in a *partition* 199 *f* 10 *i* long, and 10 *f* 7 *i* high?——Anſw. 199 10 × 10 7 = 21 *ſ* 14 *f* 10 *i* 10 *p*.

h How-many Squares in the *roof* of a houſe 18 *f* 4 *i* in front, and 37 *f* 10 *i* in depth?——Anſw. 37 10 × 18 4 = 693 7 4 + 346 9 8 (half of the ground-plot, in feet) = 1040 5 0, i.e. 10 *ſ* 40 *f* 5 *i*.

i How-many Squares in a *roof coverd with tiles*, whoſe depth, on both ſides (with the uſual allowance at the eaves) is 37 *f* 3 *i*, and the length 45 *f*?——Anſw. 37 3 × 45 = 1676 3 ÷ 10 = 16 *ſ* 76 *f* 3 *i*.

k How-many Rods of *brick-work* in a wall, 1 ½ brick thick, being 231 *f* about, and 13 *f* 4 *i* high——Anſw. 213 × 13 4 = 2840 ÷ 272 (the ſquare feet in a rod*) = 10 *r* 120 *f*.——NB. (1) When the walling is more, or leſs than 1 ½ brick thick (which is the common ſtandard meaſure) it muſt be reduc'd to that thickneſs; *by ſubtracting-from, adding-to, or multiplying-by, the* DIFFERENCE: to-wit, for 1 brick-thick, ſubtract ½ of the content; for 2 bricks-thick, add ½; for 3 brick-thick, multiply by 2; &c. (2) If the wall be of different thickneſſes, as they uſualy are in brick-houſes (being thickeſt below, and thinner every ſtory) the beſt way is to meaſure every different thickneſs by it ſelf, and to reduce all to the ſtandard thickneſs. Then add the ſeveral areas into one ſum: out of which deduct the doors and windows (meaſurd by themſelves) and the remainder will be the true area of the whole walling.

* A rod is (by ſtatute) 16 ½ feet: ſo that a rod-ſquare is 272 ¼ feet. But it being troubleſom to divide by the mixt number, it is cuſtomary with workmen to divide by 272; which gives the contents ſomething more than truth.——NB. In ſome places it is the cuſtom to meaſure by the rod of 18 feet; and, in others, by the rod of 16 feet.. So that, in the former caſe, the area muſt be divided by 324; and in the latter, by 256. V. Tables, note [n].

B *That is*, the Content of a thing, taken in feet, inches, &c. is given——by the ſolid *Foot*, in Stone [a], and Timber [b]——by the ſolid *Yard*, in Diging [c], &c.

a How-many ſolid Feet in a *ſtone*, whoſe length is 7 *f* 3 *i*; breadth, 4 *f* 5 *i*; depth 2 *f* 3 *i*?——Anſw. 7 3 × 4 5 × 2 3 = 72 *f* 0 *i* 6″ 9‴.

b How-many ſolid Feet in a piece of *timber* * 17 ½ *f* long, 23 *i* broad, and 2 *f* 7 *i* deep?——Anſw. $17\frac{1}{2} \times 23i \times 2f\ 7i = 86f\ 7i\ 9''\ 6'''$.

* *The Cuſtom* is—for *Round timber*, To gird the tree about, in the middle of the length; and folding the line twice, to take one length (or a quarter of the whole) and to account that for the true ſide of the ſquare. Then, for the length, it is accounted from the but end of the tree, ſo far up as the tree will hold half a foot girt, as they phraſe it; that is, as long as the line, twice folded, is half a foot.——For *Hewn timber*, To find the middle of the length of the tree; and there, to meaſure its breadth, by clapping two rules to the ſides of the tree, and meaſuring the diſtance between them. If the two are unequal; they add them together, and take half the ſum for the true ſide of the ſquare.

But, both theſe methods are erroneous——by *the Former*, the content being found leſs than the true, in the ratio of 11 to 14; and——by *the Latter*, more than the truth; and the more ſo, as the difference of the ſides is the greater.

C In a *cellar* 25 *f* 4 *i* long, 15 *f* 8 *i* broad, and $7\frac{1}{2}f$ deep, How-many yards of Diging?——Anſw. $25\ 4 \times 15\ 8 \times 7\ 6 = 110y\ 6f\ 8i$.

In a *vault*, Dug 9 *f* deep, $4\frac{1}{2}f$ long, 3 *f* 9 *i* broad; How-many ſolid yards?—Anſw. $4\ 6 \times 9 \times 3\ 9 = 151y\ 10i\ 6p$: i.e. $5y\ 16f\ 10i\ 6p$.

C See Decimals, and compound Multiplication.

MUTIPLICATION.

1 Mu'lt. Place the fírſt of the próduct of éach -cator únder it (2) When two;

2 Sét-down the láſt; carry fírſt. (3) Add-up áll for the total product [a].

Compendiums [b].

in Factors with Units, &c.

3 (I) Whén any númber of únits *precédes* any
dígits ; proceed thus:
4 (1) Múltiply ăs fár as the O′nes: to each-
cánd all the réſt, forward, add (2) Then,
5 Só many ás there are O′nes. (2) Then, by
ſtéps, backward, ádd the remainder [c].
6 (II) Whén any númber of units *fóllows* a dí-
git; proceed thus:
7 (1) A′dd, ſtep-by-ſtép, back the Ones (2)
Then, múltiply; ănd ſó many, back, Add [d].
8 (III) ſkípping for O′ughts interpoſd [e].
9 (IV) Whén other dígits are *intermíxt*; make
a próduct, or two more [f].

Other Digits.

10 (V) Só other dígits, by áccommodátion, are
mánagd as units [g].

Higher Figures.

11 (VI) Whén bigger fáctors approách to a
décimal dénominator;
12 (1) -cand and -cátor ſubtráct from the déci-
mal; (2) Múltiply th' rĕmainder;
13 (3) To their próduct join-báck the altérnate
ſubtrácted remainders [h].
14 (NB) Próduct than -cand—If móre places,
cárry [i]—If féwer, add ciphers [k].

COMPOUND.

15 *Compound Múltiplicátion* ŏf ſéveral námes in-
to ſeveral:
16 I′ntegers ínto ŭppĕr líne mult; add *A′liquots* [l]
.. ŏr *Cróſs*-mult [m] .. and *Mind* [n], that

17 *I'nt* into prímes give prímes [o]: *Subdiviſions* are knówn by addition [p].

a *For example:* To multiply 1234 (I) by 2: Say 2 × 4 = 8 (placing the 8 directly underneath) Then 2 × 3 = 6; and ſo-on. (II) By 6: Say 6 × 4, or rather, (putting the ſmaller number firſt) 4 × 6 = 24. Then, ſet-down 4 (the laſt figure) and carrying 2 (the firſt figure †) to the next row, Say 3 × 6 = 18 + 2 (that I carry) = 20. So, ſetting-down 0, carry (the firſt figure) 2 to the next ſtep in the work; and proceed as at firſt. (III) By 26: Say, as afore; placing the firſt figure of the product of the 6 (to-wit, 4) under 6; of 2, under 2, and ſo-on. Then, add-together the products of each figure, for the product of the whole. (IV) By 206: The procedure is the ſame as in n. III; only removing the firſt figure of the 2d product one ſtep further; to-wit, under the 2, according to the direction, lin. 1. (V) By Ciphers in the end: the procedure is as afore; only adding the ciphers (of both the factors) to the total product. Thus 200 × 400 = 80000.—Hence it appears that Multiplying (1) by 10, is only adding a cipher; ſo 1234 × 10 = 12340 (2) by 5, is halving the figures, and adding a cipher: So 1234 × 5 = 6170. (3) And, in like manner, on other occaſions.

Factors { Multiplicand 1234
Multiplicator 2
The Product 2468

II { 1234
6
7404

III { 2234
26
7404
2468
32084

IV { 1234
206
7404
2468
254204

V { 1234
20
24680

VI { S 18 7 3
6
5 11 10 2

† The ſame (or, what is equivalent) is to be done, in multiplying different denominations; as in example (VI). To operate which (1) Say, 3 × 6 *q* = 18 *q*: that is 4 *d* 2 *q*. ſet-down the 2 *q* (which belong to this row) and carry the 4 *d* to the next row) which is pence (2) Then go-on, and ſay 6 × 7 *d* = 42 *d* + 4 (that I carry) = 46 *d*: that is 3 *s*. 10 *d*. In like manner, as afore, ſet-down the 10 *d*; and carry the 3 *s*. (3) Then,

proceeding to the ſhillings, ſay 6 × 8*s* = 48*s* + 3 (that I carry) = 51*s*: Here, the eaſieſt way is to ſet-down (as in integers) the laſt figure, to-wit, 1; and to carry the firſt, to-wit, 5. So, proceed, and ſay 6 × 1 = 6 + 5 (that I carry) = 11. Set-down the odd-one; and (as in addition) carry the half of the remainder to the pounds: in which proceed as in integers——*To multiply by Addition.* V. Tabulating.

b Several of the following compendiums were the *invention* of Mr. Hatton (arithm. p. 150) who alſo gives two or three others. But, as the operation of them is pretty difficult, and the caſes occur but ſeldom, and that too out of the courſe of buſineſs; it would not conſiſt with the deſign of this treatiſe to give an account of them here.——It may not, however, be improper, here, to give a hint of *two expedients for the facilitating the operation of large ſums:* to-wit, *Logarithms*, which is made a particular ſcience; and *Neper's bones*, which is only a method of tabulating the multiplicand on ſo many (virgulae) rods, or pieces of ivory, wood, paſte-board, or the like; and may be executed with leſs trouble, and more diſpatch, by the directions here given under the head of Tabulating.

c *For example:* To multiply 652783 by 115 (I) Say, (1) 5 × 3 = 15. (2) Then 5 × 8 = 40 + 1 (that I carry) + 3 (advancing, forward, from 8 in the multiplicand) = 44. (3) Then 5 × 7 = 35 + 4 (that I carry) + 8 + 3 (going forward to the end) = 50. (II) Then, (1) Say 5 × 2 = 10 + 5 (that I carry) + 7 + 8 (going two forward, to wit, ſo many as there are ones) = 30. (2) Then, 5 × 5 = 25 + 3 (that I carry) + 5 + 2 (going 2 forward) = 37. (3) Then 5 × 6 = 30 + 3 (that I carry) + 5 + 2 (ſtill going 2 forward) = 40. (III) Then, (1) The 4 (that I carry) + 5 + 6 (going backward, at each ſtep, ſo many as there are ones from the 2 where I left-off) = 15. (2) Then 1 (that I carry) + 6 (backward again) = 7. (3) So the product turns-out (as in the example) in one line, 75070045. V. Practiſe.

$$\left\{\begin{array}{r}652783\\115\\75070045\end{array}\right.$$

d *For example:* To multiply 652783 by 511. (I) Add the places, equal to the number of units, ſtep-by-ſtep, backward: (1) Say

$$\left\{\begin{array}{r}652783\\511\\333572113\end{array}\right.$$

(1) Say 3 (the 1ſt ſtep) = 3 (2) then 3 + 8 = 11, &c. † (II) Then 5 × 3 = 15 + 1 (that I carry) + 8 + 7 (going 2 backward, to-wit, ſo many as there are ones) = 31, and ſo on.

† *Then*——If there were two, more, ones; Say (1) for the Former, 3 + 8 + 7 = 18 (2) for the Latter 3 + 8 + 7 + 2 = 20.——If an ought followd them; Say, (going-back a ſtep, the 4 ones being wrought) 8 + 7 + 2 + 5, &c.

e *For example:* To multiply 652783 by 50011. (I) Having added, by ſteps, backward, the 4 firſt figures of the multiplicand, over 0011, according to the directions of the preceding note, (II) Multiply, and ſay 5 × 3 = 15 + 1 (that I carry) = 16 + 2 [ſkiping-over 2 places (8 and 7) on account of the 2 ciphers] + 5 (on account of the 2 ones) = 23. and ſo on——And, in like manner, when the units precede.

652783
50011
32646330613

f *For example:* The following ſum may be done ſeveral

Ways	I. 1842753 9111118	II. 1842753 9111118	III. 1842753 9111118
	2060197854	204763027854	16789525285 83
	1678747983:By 911	16584777 ...By 9	By 8 ... 14742024
	16789540027854	16789540027854	16789540027854

In like manner, 26536 × 21415 will give (by 15 14 2, or 15 4 21) 568268440.

g *Thus,*

2783	2783	2783	34567
225	522	5022	6005
626175	1452726	13976226	207574835.

And others, in great variety, may, with great ſaving, and equal eaſe as at large (when the knack is familiariſd by practiſe) be done in one line: and greater combinations, in two or three.

h *For example:* To multiply 96 into 88 (I) From 100 (the decimal denominator to a number of two figures, as here) ſubtract (1) 96 (the -cand (2) and 88 (the -cator): the remainders will be 4, and 12. (II) Theſe, multiplied, give 48. (III) To this product join back 84, the remainder of 4 from 88 (the two neareſt) or of 12 from 96 (the two remoteſt) that is, alternately, the remainder of the -cand from

(I)	
Decimal denom.	100
Multiplicand.	96
Multiplicator.	88
Remaindr. fr. 100	4 12
Product.	8448

the -cator, &c: and you have the anſwer 8448, as in the margin, n. I.

i In the IId example, the product of 919 by 3 (to-wit, 2757) being 1 figure *more* than the number of places in the largeſt number to be multiplied; the firſt (to-wit, 2) is to be carried to the ſubtraction of 3 from 81, thus: 3 from 11, remains 8 + 2 (that I carry) = 10: of which ſet-down the 0; and proceed, ſaying: 1 (that I borrowd) from 8, remains 7 + 1 (that I carry) is 8.

(II)	
-cand	997
-cator	81
Remainders. .	3 919
Product . . .	80757

(III)	
-cand	9986
-cator	9994
Remainders. .	14 6
Product :	99800084

k In the IIId example, the product 84 being two figures *fewer* than the number of places in the largeſt number to be multiplied; two ciphers muſt be added, or joind back to the ſubtraction of 14 from 9494; which gives the product, as in the margin.

l *For example:* as in the margin: Where the ſeveral denominations in the multiplicand are multiplied by 9, the integers in the multiplicator: to which product are added the quotes of the multiplicand divided by the remainder of the multiplicator, to wit, . . $\frac{1}{3}$ (of a foot) for the 4 inches; and . . $\frac{1}{12}$ of that quote, for the 4 parts; 4 parts, being the $\frac{1}{12}$ of 4 inches.

	feet	*inches*	*parts*
by Aliquots	47	8	6
	9	4	4
Mult. *into* 9 *f.*	429	4	6
Divid. *by* 4 *i.* $\frac{1}{3}$	15	10	10
4 *p.* $\frac{1}{12}$	1	3	10$\frac{10}{12}$
The Product . . .	446	7	2$\frac{10}{12}$

m *For example,* as in the margin: Where——The feet, multiplied into the feet, give 423 feet——Then, croſswiſe . . the 9*f*, into 8*i*, gives 72*i*. (i. e. 6*f*.) . . and 47*f*, into 4*i*, gives 188*i* (i. e. 15*f* 8*i*)——Laſtly, the

	feet	*inches*	*p.*
by Croſs-mult.	47	8	—
	9	4	—
9 × 47 . . .	423	—	—
9 × 8 . . .	6	—	—
47 × 4 . . .	15	8	—
8 × 4 . . .	—	2	8
The Product . .	444	10	8

the inches in both the factors, multiplied one into the other, give 32 parts (i. e. 2 *i* 8 *p*).

n *In caſe of farther ſubdiviſions* *, *and larger numbers* (that cannot well be computed by the head) the eaſieſt way is to proceed (after the manner of common multiplication) as in the margin; regard being had to the inſtructions of line 16: by which it appears . . that the products are to be remov'd to the right-hand, ſo, that the degrees of each part of the operation may fall under the like in the given duodecimals . . and that 2 places given in order (as integers, and primes) to be multiplied into 2 places, produce 3 places; 3 places, into 2, produce 4; and 3, into 3, produce 5: that is, 1 leſs than the exponents, or places, or names in both the given factors.

	feet	′	″	‴	⁗
	372	11	4	—	—
	25	6	3	—	—
Into 3″			1118	10	—
6′		2232	68	—	—
25		275	100	—	—
Sum of ′	9300	107	1286	—	—
			. . . 2		
of ′	217	=2614			
		. . 10			
Product	9517	10	2	10	—

* The *ſubdiviſions* of an integer, in duodecimal arithmetic —are *diſtinguiſht* by the names of Primes ′, Seconds ″, Thirds ‴, Fourths ⁗, &c.—and may be *applied* . . in Meaſures, to Inches, Parts, &c. . . in Mony, to Shillings, Pence, &c.—only making *allowance* for the difference, according to the value of the denominations.

o *That is*, The Integer, into ′, gives ′; into ″, gives ″; into ‴, gives ‴; &c.

p *That is*, ′ into ′ [primes into primes] give ″; ′, into ″, give ‴; ″, into ″, give ⁗; &c. the product of the factors being of the denomination that the factors (*added-together*) amount-to.—NB. The great *uſe* of this operation is in meaſuring. V. Meaſuring.

PRACTISE:

Compendiums of the Rule-of-three [a].

1 IF ONE ?] [b]; The Sóught into príce [c], or its fáctors [d]; or * by′ aliquot parts [e];
2 And by the áliquots ŏf fráctions of ſought (if ány) divide price [f].
3 WHAT'LL ONE ?] [g] Thĕ príce by commódity; ŏr, if too lárge, by its factors [h] †.
4 ELSEWHERE [i]: Súbſtitute thĕ Quótes of two térms by a Cómmon-diviſor [k].
5 *At 2 ſhillings*] [l] The *únit's* place, doúbled, is Shíllings; *the reſt*, Pounds [m]:
* óf which the áliquots gĭve poúnds [n]; taking hálf, when the príce *above* 12 *d* [o].
7 *Of even ſhill.*] into half gives poúnds: doubled únits are ſhillings? [p].
8 *Réſts* (in divíding by fáctors) *to válue*: The
† laſt remainder
9 ínto -for láſt-but-one, plus reſt néxt; into -fór next, plus reſt next [q].

[a] *For the readier diſpatch of Buſineſs*, Mr Hatton (arithm. p. 99) gives us 149 rules of practiſe; and moſt Writers ſwell the matter into a very large account: but the following 3 verſes will enable one to anſwer any Queſtions, oftentimes, more readily and eaſily than all his rules, were it poſſible for any memory to retain them. ——In effect, after a due acquaintance with theſe few directions, 'Judgment and experience, as Mr Malcolm (ar. p. 542) well obſerves, will readily ſupply a variety of abridgments, depending on the ſame principle, much better than a confuſd heap of particular rules.'——It is pretty enough (to give one inſtance, out of many) to turn ells engliſh into flemiſh by adding 0, and dividing by 6

[10 bearing the ſame proportion to 6, as 5 to 3] or ells flemiſh into engliſh, by multiplying into 6, and cuting-off the laſt figure (as you are directed by Shelley, Ayres, and many of the modern writers) but, if ſuch compendiums as theſe (of which they give you not a few *) do not offer themſelves to your judgment in applying the general rules which you muſt be acquainted with: it will be very difficult to charge the memory with them; and much more ſo, to retain them: at the ſame time, that, without ſeeing the reaſon of them (which is not always very obvious) you muſt always be diſſatisfied, and uncertain. Nor, after all, is there much ſavd by them. For inſtance: To reduce 622 ells engliſh into flemiſh, the adding of 0, and dividing by 6 is not much ſhorter than multiplying into 5, and dividing by 3: which is the high road, and requires no caſting-about to find a ſhorter cut: in which there are frequently greater perplexities. And the ſame may be obſervd in all the other caſes, where the expedient is but a fanciful deviſe, to ſave a few figures.

* The largeſt collection of the beſt compendiums of this kind are in *Shelley*'s ſupplement to *Wingate*'s arithmetic, publiſht by *Kerſey*, p. 384.

b *That is*, In queſtions, wherein the conditional-term is 1 (as when we ſay, '*If one* coſt ſo-much, What will ſo-much coſt' * ?

* Or (which is the ſame) 'What comes ſo-much to, at ſo-much per pound, yard, &c.'——*NB*. The directions, in this caſe, are as follows in the text . . . *But*, in making, or examining bills by the head, it may perhaps be more obvious, and eaſy to the generality of people, to halve, or double (and ſo add, or deduct, as they ſhall find occaſion) the price of the commodity by the number of pounds, yards, &c. . . . *Thus:* (1) 13 pound of beef at 3 *d* 2 *q* per pound: Say, 13 3-pences is 6 ſix pences (or 3 *s*) and 3 *d*. And 13 $\frac{1}{2}$-pence are 6 *d* 2 *q*. Then 3 *s* 3 *d*, and 6 $\frac{1}{2}$ *d* makes 3 *s* 9 $\frac{1}{2}$ *d*. (2) 15 yards of ſilk, at 14 *s* 6 *d* per yard. Say, 15 10 *s* is £ 7-10, and 15-4 *s* is (4 times 10 *s*, and 4 times 5 *s*;) to-wit, 60 *s*, or 3 *l*. In all (to-wit, £ 7-10, and 3 *l*) £ 10 10. Then 15 6-pences are 7 *s* 6 *d*. In all £ 10 17 6. (3) 9 yards of cloth at 18 *s* 6 *d* per yard. Say 9 18 ſhillings and 6 pence is 9 *l*,

all but 9 *s* and 9 6-pences (or 4 *s* 6 *d*) to-wit, 13 *s* 6 *d*; that is (that being ſubtracted from 9 *l*) £ 8 6 6.

c I. *If the multiplicator be within the compaſs of the table*; multiply the queſtion-term (or thing-ſought) into the price. For example: (I) If one coſt 10 *s*; What will 20? Anſw. 20 (the thing-ſought) × 10 (the price) = 200 *s*; that is 10 *l*. (II) If one coſt 12 *s* 3 *d* 2 *q*; What will 134? Anſw. Multiply 134 (the commodity) into the *price*, as in the margin.

$$\begin{array}{rlrr} & 12s\text{ ————————} & S\ 16\,08 & — \\ 134\times & 3d=402\text{ ————} & 33 & 6 \\ & 2q=268=67d\text{ —} & \underline{5} & \underline{7} \\ \textit{In all} & 82l.\ 7s.\ 1d = S. & 1647 & 1 \end{array}$$

d II. *If the multiplicator be a higher number*;——Multiply the thing ſought into the factors of the price, or any two numbers, which (multiplied-together) amount to it. For example: If one coſt 12 *s* 6 *d*; What will 28? Anſw. The factors of 28 being 4 × 7: multiply the price into 4 and 7, as in the margin: the laſt product is the anſwer.

$$\left\{\begin{array}{rrr} £\text{-} & 12s & 6d \\ & & 4 \\ 2 & 10 & 0 \\ & & 7 \\ 17 & 10 & 0 \end{array}\right.$$

Note. If the multiplicator be not reſolvable into factors, take thoſe that come neareſt it;——and Add the price for the odd-one: [So, if the number were 29: 17 *l* 10 *s* + 12 *s* 6 *d* = 18 *l* 2 *s* 6 *d*] — or Multiply it into what the factors want of the multiplicator: [So, if the number were 31: 17 *l* 10 *s* + (12 *s* 6 *d* × 3) or 1 *l* 17 *s* 6 *d* = 19 *l* 7 *s* 6 *d*.]

III. *If the multiplicator be ſtill higher*; Divide the thing ſought by the even parts of the denomination, in which you would have the anſwer *. For example:—If one coſt 12 *s* 6 *d*; What will 56? Anſw. 10 *s* being $\frac{1}{2}$ of a *l*; and 2 *s* 6 *d* (which makes-up the 12 *s* 6 *d*) the $\frac{1}{8}$ of 1 *l*, or the $\frac{1}{4}$ of 10 *s* [for 8 half crowns make 1 *l*; or 4 half-crowns, 10 *s*] ſay 2 in 56 = 28 *l*. Then 8 in 56 or 4 in 28 (the quotient of 56 by 2) = 7 *l*. as in the margin, example I: in which the work ſtands in the moſt commodious form:

$$\begin{array}{ll|l} & \overbrace{\qquad\qquad}^{\text{Aliquots}} & 56\text{ lb, \&c.} \\ & & — \\ \text{I.}\left\{\begin{array}{l} s \\ d \\ £ \end{array}\right. & \begin{array}{rl} 10 & 2 \\ \left.\begin{array}{r} 2 \\ 6 \end{array}\right\} & 8 \\ \text{————} & \end{array} & \begin{array}{l} 28\text{-- --} \\ 7\text{-- --} \\ 35 \end{array} \end{array}$$

II	At 12 *s* 6 *d*.		113 lb,&c.
	s 10	2	56 10 -
	2 *d* 6	4	14 2 6
	£		70 12 6

form: the mony ſplit into aliquots, with the marks of the denominations prefixt; and the diviſors placd againſt their quotients (not fraction-wiſe) for the greater diſpatch, diſtinctneſs, and evidence.——In the IId example, [to-wit 113 lb, at 12 *s* 6 *d*] the manner of operation is the ſame: only it may not be amiſs to obſerve that *The remainder is always of the ſame name as the dividend* (V. Diviſion) and muſt be carried-on in lower names, according to the inſtructions given under the article Reduction, note [a]. Say, therefore, here (according to the directions in the former example) . . . Firſt, 113 lb ÷ 2 (2 10 *s* being 1 *l*) = 56, and 1 over: that is 1 *l*; to be divided, in a lower denomination, by the ſame diviſor, to-wit 2. Say, then, 1 × 20 (*s*, the next name) = 20: then, 20 ÷ 2 = 10 *s* . . . Again 56 *l* 10 *s* (the quotient of 113 by 2 the aliquot of 10 *s*) ÷ 4 (the aliquot of 2 *s* 6 *d*) = £ 14 2 6. for 4 in 56 *l* is 14 *l*: then 4 in 10 *s* is 2 *s*; and 2 *s* over: then 4 in (2 × 12 *d*, the next name, or) 24 *d* is 6 *d*: In all, £ 70 12 6.

* *Tables of Aliquots*, or even-parts of mony, weight, &c. are uſualy given, by writers of arithmetic, to the value of above a page; which they tell us muſt be well underſtood, and perfectly got-by-heart (V. Fiſher, arithm. p. 210, 228)——*But* common ſenſe, with the knowlege of the multiplication-table will enable any one to take the aliquots with readineſs.——*So that* it may, perhaps, be thought unneceſſary even to hint, that (I) In MEASURE . . 4 *nai* is the 4th of a *yard*; and the 5th of an *Ell* . . 7 *gall.* the 9th of a *Hogſhead*, &c. (II) In MONY . . 1 ½ *d* is an 8th of a *Shilling* . . 1 *s* 8 *d*, the 12th: 3 *s*, 4 *d*, the 6th: 6 *s* 8 *d*, the 3d of a *Pound*. (III) In WEIGHT . . 4 lb is a 7th of (28 lb, or) a *Quarter of a cw*, &c. . . 7 lb is an 8th of (56 lb, or) *Half-a-cw*, . . 14 lb is an 8th: 16 lb a 7th of a *Hundred-weight* . . 2 ½ *cw* is an 8th of a *Tun*.

f IV. *If there be fractions (or lower denominations) in the quantity of the thing ſought*; Divide the price by the aliquots thereof: proceeding in the reſt, according to the inſtructions already given. The work will ſtand as in the margin.

	Cw	*qr*	*lb*		£	*s*	*d*	*q*
	84	3	11	. . at . .	1	1	10	-
l 1 : 1 *s* =	21	*qr* 2	2		--	10	11	-
×	84	1	2		--	5	5	2
	168	*lb* 7	4		--	1	4	1
d 6 2	42	4	7		--	--	9	1
4 3	28							

Price . . . 18 6 of the fractions : 18 6 --
Total : 1852 6 : The anſwer; which, being divided by 20, gives £ 92 : 12 : 6, for cw 84 3 11.

g *That is*, In ſums, in which the Queſtion-term is one : as when we ſay : ‘ If ſo-much coſt ſo-much ; *What'll* one coſt ?

h I. *For example* : If 12 coſt 10*s* 6*d*: What will 1 coſt? Anſw. Divide (I) by 12 (the commodity) or (II) its Factors (2 × 6, or 3 × 4) according to the inſtructions given in the explication of the 2d example in note ᵉ.

			S.	10	6	--
I. By Commod:	12			--	10	2
II. † By Factors	2			5	3	--
	6			--	10	2
	3			3	6	--
	4			--	10	2

† *NB*. When the quantity of the commodity is greater ; a factor (too-big) may be divided.——For example : If 112 coſt £ 2 6 8 : What will 1 ?——Anſw. Divide by the factors 16 (or 2 × 8) and 7 ; as in the margin. V. note ᑫ.

		L. 2	6	8
112	2	1	3	4
	8	--	2	11
	7	--	--	5

i *That is*, In any queſtions of a different nature from the aforementioned, when you can, at ſight, or with a ſmall attention, diſcover the common diviſor ; that will give leſſer numbers to work with.

k *For example*.——If 7 *y* coſt 56 *l*; What coſt 35 *y*? (*NB*) The queſtion, ſtated, is : 7 *y* : 56 *l* : : 35 *y*. where it is eaſily perceivd that 7 divides both the extremes, and the quotes are 1, and 5 : So that this queſtion (1 *yd* : 56 *l* : : 5 *yd*) will have the ſame anſwer as the former ; and is found ſimply by multiplying 56 *l* by 5 ; which makes 280 *l*.——If 250 *l*. buy 548 *y*, What will 5 *l* buy? (*NB*) The extremes being both divided by 5, the quotes are 50 and 1; and the queſtion will have the ſame anſwer as this, 50 *l*: 548 *y* :: 1 *l*: which is ſolvd by dividing 548 *y* by 50: the quote is 10 *y* 3 *qrs*, 3 *nails*, and $\frac{18}{50}$.——If 27 *y* coſt 45 *l*: What coſt 63 *yards?* (NB) Here, the extremes 27 and 63 being divided by 9; the quotes are 3 and 7: and ſo the queſtion has the ſame anſwer as this: 3 *y* : 45 *l* :: 7 *y*. Again, 3 and 45 being both divided by 3; the quotes are 1 and 15: and ſo the queſtion is reduc'd to this: 1 *y* : 15 *l* : : 7 *yds*; and the anſwer 7 times 15, or 105 *l*.

l The following practiſe, by the aliquots of 2 *s*, was publiſht by Mr. *John Jones* of *Briſtol* (in 1720) as his invention; and Mr. *Snow*, in a recommendatory letter, conceives it to be intirely new.——But I find an account of it by Mr. *Mellis*, as ancient as the reign of *Edward* 6th, in Dr. *Record*'s arithmetic, p. 358.——However, as there is ſome advantage in it, I have here given it: though, in effect, there is nothing more in it than a particular application of the general directions in line 1.——For which reaſon I have added alſo the two following lines, of even ſhillings; as indicating a ſhorter way than might have offerd it ſelf from the general hints in the preceding verſes.

m Thus 748 *lb*, at 2 *s* per *lb*, is 74 *l*: and (8 being doubled) 16 *s*: for, 748 ÷ 10 (2 *s* being the 10th of a *l*) = 74 *l*. 16 *s*.

n *Thus*——For Aliquots (1) 748, at 8 *d*. Set down (as though it were at 2 *s*) 74 *l* 16 *s*; and divide it by 3 (the 8th of 2 *s*) you have the anſwer £ 24 18 8. (2) 721, at $1\frac{1}{2}$ *d*. Say 72 *l* 2 *s* ÷ 8 (the number of $1\frac{1}{2}$ *d* in a ſhilling) = £ 9 0 3 ÷ 2 (the number of 1 *s* in 2 *s*) = £ 4 10 $1\frac{1}{2}$.——for Aliquants (1) When the given price conſiſts of odd farthings, or is under 2 *d*; work it firſt at 3 *d*, and take the parts. Thus 210, at 1 *q*. 21 *l* ÷ 8, the 3 *d* in 2 *s*) = £ 2 12 6 ÷ 12 (the 1 *q* in 3 *d*) = S 4 4 ½. (2) When the given price is above 2 *d*, but an aliquant part; you muſt be content to ſplit it into aliquots. Thus 745, at 5 *d*. 74 *l* 10 *s* ÷ 6 (the 4 *d* in 2 *s*) = £ 12 8 4 ÷ 4 (the 1 *d* in 4 *d*) = £ 3 2 1; in all, £ 15 10 5.

o *For example:* 719, at $15\frac{1}{2}$ *d*: For the 12 *d* take half of 71 *l*. 18 *s*; to-wit, 35 *l* 19 *s*: then, of that, for 3 *d*, the 4th; and of that, for 2 *q*, a 6th. The anſwer will be £ 46 8 8 2.

p *For example:* 379, at 18 *s*. Say 9*d* (the half of the price) × 379 = 3411; that is (the unit place being doubled) 341 *l* 2 *s*.——In caſe of *an odd ſhilling:* a 20th of the commodity added to the price of the Even, gives the total. Thus 379 at 19 *s*. To 341 *l* 2 *s* (the price, at 18 *s*) add 18 *l* 19 *s* (the 20th of 379) the total will be 360 *l* 1 *s*.

q *That is:* To value the remainders of a diviſion by the factors of any given number——Multiply the laſt remainder into the laſt-diviſor but-one; to-wit, that overagainſt it, in the poſition I recommend (V. Reduction note).——To the product add the preceeding remainder.——The ſum thereof multiply into the next preceeding diviſor: And ſo on, till you have gone-up, through all the diviſors, and remainders, to the firſt remainder. Thus, in the example annext, the value of the particular fractions, denoted by the three remainders (1 4 5) will be found $\frac{103}{126}$, as againſt the laſt dividend. For 5 (the laſt remainder) × 6 (the preceding diviſor) + 4 (the preceding remainder) × 3 (the preceding diviſor) + 1 (the preceding remainder) = 103; for a numerator, to 106 (the product of the factors, or) the denominator.——

Diviſors		*Dividend*	*Rem^rs*
	126	3428689	1
factors	3	1142896	4
	6	190482	5
	7	27211	= $\frac{103}{126}$

NB. This holds-good (I) In all the Changes, or tranſpoſitions of the diviſors, and varieties of remainders: as will appear . . by dividing the above-ſum by the diviſors in the margin . . and operating the remainders, as before explaind. (II) In Applicate numbers: as may appear by the ſolution of the two following queſtions. (I) In 3476589426 *gr*. How-

Diviſors			*Remainders*		
3	7	6	1	6	4
6	3	7	1	2	5
6	7	3	1	3	2
7	3	6	5	2	4
7	6	3	5	2	2

factors of 5760 the number of *gr*. in a *lb*.			
	3476589426	*gr*	2
24 { 4	869147356		4†
6	144857892	*dw*	12
20	7242894	*oz*	6
12	603574	*lb*	‡

many *lb* &c? (2) What is the value of 1 yard of cloth, 48 yards of which coſt £ 15 10 4 ? (NB) The value of the remainders, in the abbreviated form, being found by the method aforementiond, appear to agree with the work, at large, as performd by reduction and diviſion, in the margin, underneath the operations by the factors of 5760 and 48.

Gr. 3476589426(5760 *gr* in *lb.*
20509426 603574 *lb.*
337928
4261
—3 12* *oz.* in *lb.*
38232(6 *oz.*
,3672
20 *dw.* in *oz.*
73440(12 *dw.*
15840
,432
24 *gr.* in *dw.*
103680(18 *gr.*
4608,0

factors of 48						
£	15	10	4	0	—4
6	2	11	8	2	—2
8	--	6	5	2	—$\frac{16}{48}$

£ . . 15 10 4
20
s. . . 310
12
d. . 3724
4
q. 14896

14896(48
4,1 *q.* 310—2
d. .77—5
s. . .6—
i. e. (by reduction)
S 6 5 2 $\frac{16}{48} = \frac{1}{3}$.

† Here, 4 (the laſt remainder of the diviſion by 6, the latter factor of 24) × 4 (the preceding diviſor) + 2 (the firſt remainder) = 18 *gr.* as in the valuation of the diviſion, at large, underneath.

‡ Here, 6 × 20 + 12 × 6 + 4 × 4 + 2 = 3186 : agreeable to what appears from the remainder of the diviſion at the top of the page.

* Here, 3186 (the remainder of the diviſion, that reduces *gr* to *lb*) × 12 (the number of (the next inferior denomination, to wit) *oz* that make 1 *lb*) = 38232 : which, divided by 5760 (the original diviſor) gives 6 *oz.* And ſo on, according to the directions given in the article Reduction, note [a].

PROGRESSION.

1 PR'OG. AR : a ſéries ŏf nŭmbers *by équal additions* increaſing [a].

2 *Sum of a ſéries* is áll into th' ſúm of the extremes by 2[b].
3 *Difference of ſéries* ĭs thát o'th' extrémes, by the terms, leſs unit[c].
4 *Nûmber of térms:* firſt from láſt: / reſt, by dífference, gĭves All-but-unit[d].
5 PR′OG. GE: a ſéries ŏf númbers *by óne common fáctor* increaſing[e].
6 *Láſt of a ſéries* tŏ find, conceíve arithmétical Exponents[f]:
7 twó of which, ádded, denote the próduct o'th' térms correſponding[g].
8 Then for the *ſúm of the ſéries:* thĕ laſt term ínto the ratio:
9 ánd, from the próduct, the fírſt: the remáinder by th' rátio leſs unit[h].

a *Thus*, 1 2 3 4, or 4 3 2 1 (differing by 1) and 2 4 6 8, or 8 6 4 2 (differing by 2) are ſaid to be in arithmetical progreſſion.——*Note*. In arithmetical progreſſion (I) are to be obſervd the two extremes: to-wit, (1) the firſt term, *a*. (2) the laſt term, *l*. (3) The number of terms, *n*. (4) The common difference, *d*. (5) The ſum of all the terms, *s:* (II) any three of which being given, the other two may be found: as may be ſeen in a ſeries of 20 propoſitions in Oughtred's clavis math. 29. 4. See alſo Malcolm, arithm. p. 233, and p. 243; where he gives ſeveral problems, wherein two things only are given, to find the other three. (III) The chief and moſt uſeful of the problems are thoſe which relate to the differences, or the ſum of the ſeries, &c. as follows in the text.

b *For example:* [Given a l n; to find S] How far does he go, that carries, one-by-one, into a baſket, 100 ſtones, a yard diſtant from one another; the firſt of 'em a yard from the baſket,——Anſw. $\left[\frac{a+l\times n}{2}=S\right]$ 10100 yards. For 100 (the number of terms) × 202 (the ſum of the extremes; to-wit, 1, and 100 doubled, on account of going backward and forward) = 20200 ÷ 2 = 10100 *; that is 5 miles and $\frac{3}{4}$, wanting 20 yards.

* Sir *Jonas Moore* makes the diſtance run but 10000 yards; which is too little by 100 yards. V. Moore's arith. p. 324.

c *For example:* [Given a l n; to find D] One had 12 children, that differd alike in all their ages: the youngeſt was 9 years old, the eldeſt was 36 $\frac{1}{2}$: What was the difference of their ages, and the age of each?——Anſw. $\left[\frac{l-a}{n-1}=D\right]$ 27.5 (the difference of the extremes; to-wit 36.5 —9) ÷ 11 (the number of the terms leſs 1) = 2.5, the common difference requird. Conſequently, the age of the youngeſt but one is (9 + 2.5, i. e.) 11.5: of the next (11.5 + 2.5: i. e.) 14: and ſo of the reſt.

d **For example:** [Given a l d; to find N] A man going a journy, his firſt day's travel was five miles; his laſt day's travel was 35 miles: he increaſt his journy, every day, three miles: How-many days did he travel? (Anſw.) 11 days. For, 5 (the firſt term) — 35 (the laſt term) = 30 ÷ 3 (the difference) = 10 + 1 = 11; the number of days ſought.

e *Thus,* 2 4 8 16 (increaſing by one common factor, calld the ratio) are ſaid to be in geometrical proportion.

f The *exponents*, or indices, are a ſeries of numbers in arithmetical progreſſion, begining with 1 *, and the common difference 1.

* *NB.* If the ſeries of geometrical proportionals begins with 1, the exponents begin with a cipher.

g *For example:* [To find any term, whoſe diſtance from the firſt is aſſignd; without producing the whole ſeries] A horſe was offerd to be ſold at a farthing a-nail, and double it: that is, 1 farthing for the firſt, 2, for the 2d; 4, for the 3d, &c. in geometrical progreſſion: the number of nails, 7 in each ſhoe. What would have been the price of the horſe at that rate? Anſw. (1) Firſt 0 1 2 3 4 5 indices or exponents, in arithm. progr. 1 2 4 8 16 32: farthings in geometrical progreſſion. (2) Then [theſe leading terms being found with their exponents, ſay] 5 + 5 = 10: therefore, 32 × 32 = 1024. (3) Then 10 + 10 = 20: therefore, 1024 × 1024 = 1048576. (4) Again, 4 + 3 = 7: therefore, 16 × 8 = 128.

(5) And 20 + 7 = 27: therefore, 1048576 × 128 = 134217728: which is here be accounted the 28th, and laſt term; becauſe the 1ſt term in the ſeries is 1, which does neither multiply, nor divide.

h *For example:* In the preceding ſeries, the laſt term is 134217728; being the number of farthings to be paid for the laſt nail. By this, with the 1ſt term and ratio, is found the ſum of all the ſeries, as follows: 134217728 (the laſt term) × 2 (the ratio) = 268435456— 1 (the 1ſt term) = 268435455 ÷ 1 (the ratio, leſs unit) = 268435455; that is, £ 279620. 5. 3. 3.

PROOFS

1. OF THE OPERATIONS:

or, the means of being ſatisfied that they are rightly performd; are as follows:

1 PRÓVE by a cáreful review; 'tis the *ſáfeſt:* the *réadieſt*, as follows:

2 SÚB.] *right*; when -hénd and remáinder (together) máke up the compound [a].

3 ADD MULT DÍV] add the dígits togéther and cáſt-out the nines: then

4 *Right*; if remáinder of Fácits agreés with remáinder of factors [b],

5 Múltiplied ĭn Múl [c]: -ſor and quótient in Dív; to which ádd the remainder [d].

2. OF THE RULES:

6 RÚLES *Prove* by Várying the wórk in a Dífferent, ór in the Same rule [e].

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[a] *For example:* 234 — 34 = 200.——Proof: 34 (the ſubtrahend) + 200 (the remainder) = 234; the ſame as

the compound number, from which 34 was ſubtracted: therefore the work is right.

b Note——*In Addition,* the particulars may (not improperly) be calld the factors; and the total, the facit.——*In Diviſion,* the diviſor and quotient multiplied, with the remainder added, make the dividend; and may, therefore, be conſiderd as factors.——To *Exemplify:*

In Addition: $3 + 9 + 15 + 7 + 23 = 57$.——Proof: Say (in the factors, or particulars) $3 + 1 + 5 = 9$: then $7 + 2 = 9$; and 3 remains. Then (in the facit, or total) $5 + 7 = 12$: the 9 caſt-out, there remains 3; the ſame as the remainder of the factors: therefore the work is right.———*In caſe of Different Names* (1) either Add the Columns upward and downward (2) or Divide a long ſum into parcels; and, taking the ſeveral totals on a bit of paper (laid juſt underneath each parcel) add them together at the foot of the account.

c *In Multiplication:* $3685723 \times 56 = 20640048$——Proof: The nines being caſt-out of the multiplicand, there remains 7; out of the multiplicator, there remains 2: then 2×7 (the remainders of the factors) $= 14$: out of which the 9 being caſt, there remains 5: with which agrees the remainder of the facit, or product, the nines being caſt-out: therefore the work is right.

d *In Diviſion:* $3268 \div 67 = 48$.——Proof: The nines caſt out of the diviſor, there remains 4; out of the quotient, there remains 3. Then 3×4 (the remainders of the factors) $= 12$: out of which the 9 being caſt, there remains 3: then (this) $3 +$ (the digits in the remainder, to-wit) $5 + 2 = 10$; out of which the 9 being caſt, there remains 1: with which agrees the remainder of the facit, or dividend, the nines being caſt-out: therefore the work is right *.

* Having thus explaind the manner of proof by caſting-out the 9 (nines) it may not be improper to ſhow the truth of it. In order hereto, it is to be premiſd that the figure, that ſtands in any place of a number (taken in its ſimple value) is equal to what will remain, after 9 is taken-out of the complete value, as oft as poſſible; that is, after all the 9's (containd in it) are taken-away. *For example,* If all the 9 (nines) containd in 700 are taken-away, there remains the ſimple number 7. The demonſtration of this lemma ſtands thus; Any figure,

ſtanding in any place of a number, is equal to ten times the value of the ſame figure, in the next lower place (by what has been ſhown in Notation) that is, equal to 9 times, + 1 time that value (becauſe $9 + 1 = 10$) But 9 times any number is a preciſe number of 9 (nines) which being taken-away, there remains once the value of it in that next place: and this, again, is equal to 9 times, + 1 time the value of the ſame figure in the next lower place; and the 9 times being taken-away, the 1 times remains: and ſo on, till you bring it down to the place of tens; where it is equal to 9 times its ſimple value, + once the value: and, the 9 times taken-away, there remains the ſimple value. And thus we have ſuppoſd all the 9's to be taken out of it; and, conſequently, the lemma is true.——But, to this proof it is objected, that a wrong operation may appear to be true. Anſw. (1) This muſt be own'd. for if we change the places of any two ſignificant figures in the ſum, it will ſtill appear to be right; there being the ſame exceſs of 9 (nines) where there are the ſame figures, whatever order they ſtand in. (2) But, then, conſider: a true ſum will always appear true by this proof (for that is demonſtrated) and, To make a falſe ſum appear true, there muſt be at leaſt two errors; and theſe oppoſit to one another: that is, one figure greater than it ought to be, another as much leſs: and, if there are more than two errors, they muſt always balance among themſelves: that is, the ſum of the figures, that are greater than they ought to, be, muſt always be equal to the ſum of the figures that are deficient: elſe it is plain, a falſe ſum will not appear to be right. But, now, if we conſider what an exceeding great chance there is againſt this particular circumſtance of the errors; and how ſimple the proof-work itſelf is: we may truſt to this proof as ſafely as to any other.

Thus, Practiſe may be provd by the rule-of-three, or by a different working in the rule of practiſe; as in the following example:

At 5*s*. 8*d*. . 756*lb*			*Or* . . . . 756 *lb*.				*Or* 1 : 5*s*. 8*d* : : 756	
		5					12	
			*s*4	5	151	4		
s		3780	1	4	37	16	68*d*.	
*d*4	3	252	*d*6	2	18	18	756	Rule-of-three
4	3	252	2	3	6	6		
							6048	
s		4284	£		214	4	4536	
£		214:4					*d*. 51408(12	
Practiſe							4284	
							£ . . . . . 214 : 4	

PROPORTION.

1 PROP.—*Continued*, Progréſſion is calld--*Diſjúnct*, Rule-of-three—and

2 *Cónjunct*, when (ín the ſame quéſtion) are compard Coíns, Weights, and Meaſures.

3 RULE. *This:* I'nto each óther the I'ſs, and the Mákes; for diviſion [a]:

4 *Hów-many Láſt* make the gíven o'th' Fírſt? -for, the I'ſs; -dend the Reſt make [b].

5 *Hów-many Fírſt* make the gíven o'th' Láſt? -dend, the I'ſs with the Queſtion [c].

[a] *To anſwer queſtions of this nature* (in the readieſt manner, without repeated ſtatings) by *Diviſion* only; the Ifs or (ſuppoſitions) are to be enterd in one column; and the Makes (or what they are ſuppoſd equal-to) in another. Then, each rank, multiplied continually, give the dividend and diviſor, according to the directions in the two following caſes.

[b] *For example:* If 10*l* at London make 9 at Amſterdam; and 90 at Amſterdam are equal to 112 at Tholouſe: How-many at Tholouſe are equal to 50 at London? —Anſw. The Ifs 10 x 90 = 900 for the diviſor: the Reſt (to-wit, the Makes with the queſtion) 9 x 112 x 50 = 50400

= 50400, for the dividend. Then, 50400 ÷ 900 = 56 *l.* at Tholouſe, equal to 50 at London.

C *For example:* If 100 *l* Engliſh, make 95 Flemiſh; and 19 Flemiſh make 35 at Bologna: How-many Engliſh are equal to 50 at Bologna?——Anſw. the Iſs, with the queſtion, 100 × 19 × 50 = 95000, for the dividend: the Reſt 95 × 35 = 2375, for the diviſor. Then, 95000 ÷ 2375 = 40 *l* Engliſh, equal to 50 at Bologna. V. Exchange.

QUANTITIES.

of goods, &c. nearly (at leaſt) determinate in

NUMBER, WEIGHT, MEASURE.

The following liſt, whatever defects or errors there may be in it (which the curious will be ſo good as to appriſe me of) will not (I preſume) be unacceptable: as, thereby, a ready recourſe may be had to ſuch notices as frequently occur, in reading, and in the way of buſineſs; for the explication whereof little help is to be had from dictionaries, or books of arithmetic.

AEM, *Awme*, *Awame:* *Gallons:* (of wine) 35 from Antwerp: 40, from the Rhine: 50, from Dordrecht.

ANKER: ¼h of an aem.

BAG:——*Buſhel:* 1, of lime——*Hundred-weight:* 1 ¼ to 1 ¾, of hops: 1 ¼ to 3, of pepper: 2 to 3, of ſafflower: 2 to 4, of goat's-hair: 2 to 7, of ſumach: 2½ to 4 ¼, of cotton-yarn: 3, of almonds: 3 to 4, of aniſeeds: 4, of currants.

BALE——*Bolts:* 100, of lyons and paris thread——*Hundred-weight:* 1 to 4, of perſia raw ſilk: 1 ½, of cochineal, indigo: 2, of cardamoms, thrown ſilk: 2 ¼, of ſpaniſh wool: 3, of carraway-ſeeds: 3 to 4, of cotton-yarn: 6, of ſafflower: , of feathers: , of cork.——*Pieces:* 3 to 4, of lockrams: 20, of boultels: 22 ½, of bevernix, and holmes fuſtians——*Reams:* 10, of paper: 100, of unbound books.

BAND:——*Strikes:* 10, of eels.

BANDLE—*Feet:* 2 in length; an iriſh meaſure.

BARREL—*Buſhels:* 3, of apples, pears—*Dozen:* 10, of candles—*Gallons:* $31\frac{1}{2}$, of oil: 32, of herrings [a], ling: 42, of eels [a], mum, ſalmon [b]—*Hundred-weight:* $\frac{7}{8}$ to $2\frac{1}{2}$, of figs: 1, of gunpowder, lippora raiſins: 1 to $1\frac{1}{2}$, of quickſilver: 2 to 3, of ſpaniſh-tobacco.—*Number:* 300, of black or white plates: 1200, of ſtock-fiſh. V. Hundred.—*Pounds:* 16 (the little) 30 (the great) of anchovies: 100, of gun-pouder for ſhips: 120, of candles: 200, of barillia, oat-meal: 224, of butter: 240, of ſoap: ..... [a] By 2 *H.* 6. 11, the barrel of herrings and eels ſhall contain 30 gallons. [b] By 2 *H.* 6. 11, the barrel of ſalmon ſhall contain 84 gallons.

BASKET—*Buſhels:* 2 of medlars—*Pounds:* 20 to 50, of aſa-fœtida.

BAVIN, V. Wood.

BILLET: *Feet* (of wood) 3, in length: whereof there ſhould be 3 ſorts (1) a Single billet, 7 inches, about (2) a Caſk, 10 inches (3) a Caſk of two, 14 inches. V. Wood.

BIND—*Strikes:* 10, of eels.

BOLT—*Ells:* 28, of poldavies.—   , of lyons and paris thread.

BOX—*Groſs:* 2, of rings for keys—*Hundred-weight:* 1 to 2, of quickſilver.—*Pounds:* 14, of prunelloes.

BUNCH—   , of rheniſh glaſs.

BUNDLE—*Feet:* 3 (about, at the band) of baſket-rods.—*Load:* $\frac{1}{60}$, of bulruſhes—*Number:* 10, of necklaces, glover's knives, harneſs-plates, baſs-ropes: 16, of ſets of inſtruments for barber-ſurgeons: 100, of laths, 5 foot long: 120, of laths, 4 foot long—*Skains:* 20, of hamburgh yarn.

BURDEN—*Pounds:* 180, of gad ſteel.

BUTT—*Gallons:* 84, of ſalmon—*Hundred-weight:* 15 to 22, of currants.

CADE—*Number:* 500, of red herrings: 1000, of ſprats.

CAG, *Kegg*—*Gallons:* 4 to 5, of brandy, ſturgeon.

CANE—*Inches:* 12 or 14, of rods of lead for glaſiers.

CANISTER—*Pounds:* 75 to 112, of tea.

CAROTEEL—*Hundred-weight:* 3, of mace: 4 to 5, of cloves: 5 to 9, of currants: 6 to $7\frac{1}{2}$, of nutmegs.

CARRAGE—*Buſhels:* 64, of lime.

CARUCATA, V. Hide.

CASATA, V. Hide.

CASE—*Feet:* 120, of

normandy-glaſs.—*Number:* 5, of recorders: 120, of window-glaſſes.——*Pieces:* 21, of hollands linen.—*Tables:* 24, of ratcliff crown-glaſs: 25, of french-glaſs: 35, of newcaſtle-glaſs.

Cask--*Hundred-weight:* 1¾ to 3, of tobacco: 2, of wheat-flour: 3, of almonds: 8 to 11, of ſugar.

Chain—*Links,* 100; or 4 ſtatute-perch: of which, one in breadth, and 10 in length, make an acre.

Chaldron, *Chaudron*—*Bolls:* 16, of corn——*Buſhels:* 32, of corn: 36, of coals——*Number:* , of grind-ſtones.

Chest——*Flaſks:* , of florence-wine—*Feet:* 200 to 300, of glaſs—*Hundred-weight:* 1½, of cochineal: 1½ to 2, of indigo; 5 ſcore to the *cw:* 2½ to 3, of caſtile-ſoap: 3½, of benjoin, iſing-glaſs: 3½ to 4, of myrrh: 10 to 15, of ſugar.

Chief—*Ells:* 10, of fine linen, ſilk: 14, of fuſtian.

Cord—*Feet* (of wood) 8 long, 4 broad, 4 deep. (NB) That, calld the 14-foot cord, is to be 14 feet in length; 3, in breadth; and 3, in depth. V. Wood.

Dakir (51 H. 3) a Dicker.

Denariata—*Acre:* 1, of land.

Dish—*Cubic-inches:* 1073.52 (near 4 corn-gallons) of lead-ore: which, if pretty good, will yield about 3 *hundred-weight* of lead.

Dozen——13, of tand calf-ſkins—14, of rolls.

Dupper——*Hundred-weight:* 1, of roman vitriol: 1 to 1½, of tincal.

Fagot——*Feet:* 3 (in length) of wood; and, at the band, 24 inches about, beſide the knot.—*Pounds:* 120, of ſteel.

Familia, V. Hide.

Fangot——*Hundred-weight:* 1¼ to 2, of raw-ſilk of aleppo: 1½, of thrown-ſilk of naples: 1¾ to 2, of yarn.

Fardel—*Yard-land:* ⅛, of land.

Farding-deal, *Farundel* [a], *Acre:* ¼, of land....

Fatt, *Vat*——*Bundles:* 210 to 221, of yarn.—*Buſhels:* 8, of corn.—*Hundred-weight:* 3¼ to 4, of iſing-glaſs: 5, of briſtles: 20 to 25, of wire.——*Maunds:* 1½, of unbound books——*Pieces:* 200, of narrow germany-linen.

Floor—*Feet* (of wood) 18 long, 18 broad, 1 deep.

Fother, *Foder*—*Hundred-weight:* 19½ (of lead) among the plumbers: 21, at newcaſtle: 22, at ſtockholm: 22 ½, at the mines.

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[a] In a ſurvey-book of the manor of weſt-ſlapton, in com. devon, is enterd thus: *A B* holds 6 *farthings* of land, at 126 *l*, per annum.

Frail—*Pounds*, 75 to 100, of raiſins.

Furr——*Pains*: 4, of budge-poults.

Goad—*Ell engliſh*: 1, * of welſh frizes, and frizados.—*55 inches, Hayes, negot. magaz. p. 206.

Gunny———*Hundred-weight*: ¾, of cinnamon: 1, of aloes-epatica, benjoin: 1 to 1 ½, of turmeric: 1 ¼, of ſalt-petre.

Hide, *Hyde*, *Hyda*, *Carucata*, *Caſata*, *Familia*, *Manens*. *Manſum*, *Ploughland*, *Sullinga*—*Acres*: 100, or 120 of land.

Hundred—*Bags*: 25, of lime—*Bundles*: 70, of pipe-hoops: 90, of hogſhead hoops: 120, of barrel and kilderkin hoops: 180, of pink or firkin hoops.—*Ells*: 120, of canvas (except quilted, ſtrip'd, and tufted) and linen-cloth—*Inches*: 100 times 72 (or a piece of 12 foot by 6 inches) of wood, in carpentry.—*Number*: 80, of pales ſix-foot long: 120, of anchor-ſtocks, balks, barlings, barrel-boards, battens, cabbage-plants, capravens, clapholt, deals, eggs, codcole- ſtock-fiſh, handſpikes, headings (for barrels, pipes, &c.) red herrings, lathes of three foot long, morkins, oars, pack duck, pales of four foot long, ſack-cloths, coney- lamb- ſheep-ſkins, boom-ſpars, bow-ſtaves, wainſcots, walnuts: 124, of haberdine, or ling——*Tuns*: 14, of ſalt at amſterdam. V. Quintal.

Great Hundred—*ſmall-Hundred*: 24, of clapboard.

Hyde, V. Hide,

Jarr—*Gallons*: 18 to 26, of oil—*Pounds*: 52, of wheat: 100, of green-ginger.

Ingot——*Pounds*: to , of bullion, or melted gold and ſilver.

Keg, V. Cag.

Kintal, V. Quintal.

Knight's fee—*Hides*: 12, of land: or, ſo much inheritance as is ſufficient to maintain a knight, with ſuitable retinue: which, in Henry IIId's days, was reckond at 15 *l*: but Sir T. Smith rates it at 40 *l*.

Last—*Barrels*: 12, of pot-aſhes, cod-fiſh, white-herrings, oat-meal, pitch, tar: 24, of gun-powder—*Cades*: 20, of red-herrings—*Dickers*: 20, of leather—*Dozen*: 12, of hides—*Hundred-weight*: 1 ½ to 4, of cotton wool: 17, of flax—*Number*: 1000, of ſtock-fiſh——*Pair*: 3, of dog-ſtones—*Pounds*: 384, of any commodity in ſcotland: 1700, of feathers, flax.—*Quarters*: 9 of meal:

10,

10, of corn, rape-ſeed—*Tuns:* 12, in eſtimating the contents of ſhips.

Librata—*Acres:* 240, of land: or, 20 ſolidatas.

Load—*Bundles:* 60, of bulruſhes—*Buſhels:* 40, of corn, lime—*Diſhes:* 9, of lead-ore.——*Feet ſquare:* 150, of four-inch plank: 200, of three-inch: 300, of two-inch: 400, of one-inch-and-half: 600, of one-inch. —*Number:* 50, of fagots: 100, of bavins: 500, of bricks: 1000, of tiles.——*Pounds:* 175, of lead.——*Truſſes:* 36, of hay. V. Fother, Poke, Seam, Tun.

Manens, *Manſum*, V. Hide.

Mantle, V. Pain.

Mark—*Ounces* (averd.) 8, of french copper- gold- ſilver-thread: $11\frac{1}{2}$ (venice-weight) of french gold- and ſilver-thread.

Mast——*Pounds* (troy) $2\frac{1}{2}$ of amber, cologn gold- and ſilver thread.

Maund—*Bales:* 8, of unbound books.

Nest—*Cheſts:* 3, of cypreſs-wood coffers.

Nook, *Nocata*——*Acres:* $12\frac{1}{2}$, of land.

Obolata—*Acre:* $\frac{1}{2}$, of land.

Ox-gang, *Oſkin*—*Acres:* 15, of land: or, as much as an ox can plough in a ſeaſon.

Pack—*Number:* 52, of cards: 20000, of teaſels—*Pounds:* 240, of wool: 480, of iriſh yarn.

Packet—*Number:* 250, of needles.

Pain, *Mantle*—— , of furs.

Peisa, V. Weigh.

Piddle—— , of land.

Piece——*Ells:* 13, of lawns: 106, of lockrams: 120, of moſt linens.——*Yards:* $2\frac{1}{2}$, of carpets of tunis: 4 to 8, of fileſia-lawns: $7\frac{1}{2}$ of ſcamoty: 10, of checks: 12 to 13, of devon-dozens (wt. 13 lb) ordinary peniſtones (wt. 28 lb): 13 to 14, of northern dozens ſingle (wt. 35 lb. frized) ſorting peniſtones (wt. 35 lb. unfrized): 15, of baffins, bombaſins, buſtians, carrels, dornix, fuſtians, raſhes, ſackcloths, ſayes: 17 to 18, of narrow yorkſhire kerſeys (wt. 22 lb): 24, of broad-cloth (the ſhort piece) friſados, hounſcot ſayes, newberry-whites, and other kerſeys of like make: 25, of ſpaniſh-cloth (wt 43 lb): 28, of ſorting hampſhire kerſeys (wt. 32 lb): 30 (the double piece) of fuſtians, cloth-ſerges: 32 (the long piece) of broad-cloth: 36, of caddas.

Plough-land: ſo much as may be tilld with a ſingle plough. V. Hide.

Pocket, *Sarplar*, *Serpliathe*

pliathe—*Pack:* ½, of wool —— , of hops.

Poke—*Hundred-weight:* 20, of wool: calld (in ſome places) a *load*; being a wagon-load.

Pot——*Gallon:* ½, in guernſey and jerſey.

Puncheon——*Hundred-weight:* 10 to 12, of prunes.

Quadrantata—*Acre:* ¼, of land.

Quintal, *Kintal*, *Hundred-weight*—*Buſhels:* 25, of lime——*Pounds:* 75, at leghorn: 100, of cloves, cochineal, fiſh (at newfoundland, and in the ſtreights) ginger, indigo, mace, nutmegs, pepper, ſugars (in the engliſh ſettlements in america) braſil- ſt chriſtopher's- ſpaniſh- verinus- tobacco, mohair- raw- linen-yarn: 120 (calld *long-weight*) of cheeſe (in cheſhire, derbyſhire, lancaſhire, leiceſterſhire, ſhropſhire, ſturbridge-ſair) coarſer metals[a], and iriſh yarn . . . .

Ration--*Pecks:* 9 ¾: *or*, a days allowance of bread or forage, for man or horſe.

Ring——*Number:* 240, of clap-boards.

Rod—*Number:* (of Candles) 12, of ſix in the lb: 16, of eight in the lb: 24, of twelve in the lb.

Sack——*Buſhels:* 3, of coals: 4, of corn: 5, of ſalt——*Hundred-weight:* 1 ½ to 4, of cotton, wool——*Stone:* 26, of ſheep's wool (14 lb. to the ſtone: but, in Scotland 24, of 16 lb. to the ſtone).

Sarplar, V. Pocket.

Saume——*Pounds:* 315, of quickſilver.

Score—*Chaldrons:* 21, of coals.

Seam——*Buſhels:* 8, of malt——*Horſe-load:* 1, of wood——*Pounds:* 120, of glaſs.

Semibole——*Pipe:* 1, of wine.

Seron——*Hundred-weight:* 2, of almonds: 2 ½ to 3 ¾, of caſtile-ſoap: 3, of barillia: 3 to 4, of aniſeeds.

Set——*Number:* 5, of recorders: 24, of alphabets.

Serpliathe, V. Pocket.

Shid *of wood*——*Feet:* 4 in length: and in girth, according as they are markt. If they have but 1 notch, they are to be 16 inches about: if they have two notches, they are to be 23: if 3, 28: if 4, 33: if 5, 38.

Shock——*Ells:* 13, of lawn——*Number:* 60, of ſoap-boxes, canes, trays——*Yards:* 4 to 8, of ſileſia-lawns.

Skin—*Hundred-weight:* ¾ of cinamon.

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[a] It is alſo calld the *ſtannary-hundred*; tin being hereby weighd to the king's farmers.

SOLIDATA—*Acres:* 12, of land: or 12 denariatas. V. Farding-deal, Obolata.

SORT——*Dozen:* 4, of balances——*Ells:* 106, of lockrams: 120, of ſeveral linens.

STACK—*Feet* (of wood) 3 long, 3 broad, 12 high.

STAND--*Hundred-weight:* 2½ to 3, of burgundy-pitch.

STICK——*Rods:* 30, of candles.

STOOK——*Sheaves:* 12, of corn.

STRIKE——*Number:* 25, of eels.

SULLINGA, V. Hide.

SUM—*Number:* 10000, copper- harneſs- roſe- ſadlers- ſprig nails.

THRAVE—*Sheaves:* 24, of corn.

TRUG——   , of corn.

TRUSS——*Pounds:* 56, * of hay: , of forage, as much as a trooper can carry on his horſe's crupper.... * 60, in july and auguſt.

TUB—*Hundred-weight* 3 to 4, of vermillion——— *Pounds:* 56 to 86, of camphor: 60, of tea.

TUN—*Bales:* 5, of feathers: 8, of paper: 10, of cork——*Barrels:* 2½, of brandy: 3, of ſyrop: 4, of prunes——*Buſhels:* 20, of cheſtnuts, wheat, and other grain: 42, of ſalt. V. Buſhel, in Tables, note [n].——*Dozen:* 1, of planks: 2, of walnut-tree tables——*Feet-ſquare:* of timber. V. Load. ——*Gallons:* 236, of ſweet oil: 252, of whale oil.——*Pounds:* 1709, of barley: 2100 to 2240, of rye: 2200 to 2500, of wheat: 2000, the ſea-tun, by which the contents of a ſhip are eſtimated...... Corn is uſually ſold, in England, by the *quarter:* and 5 quarters is commonly reckond to a tun in freight.

VAGA, V. Weigh.

VAT, V. Fatt.

WEIGH, *Wey*, *Waga*, *Vaga*, *Peiſa.*——-*Bunches:* 60, of rheniſh glaſs——*Caſes:* 60, of window glaſs ——*Pounds:* 224, (of cheeſe) by 9 *H* 6. 8: 248, in eſſex: 256, in   : 300, in   : 336, in ſuffolk (of bay-ſalt)——*Quarters:* 6, of barley and malt: 5, of other grain.

WINCH——   :   , of cable-yarn.

* WOOD, is aſſizd into Shids, Billets, Fagots, Falwood, and Cord wood. V. Shid, &c.

YARD-LAND——*Acres:* 15, at wimbledon in ſurry: 20, in moſt other places: 24, 30, 40, in ſome.

REBATE.

1 DISCOUNT[a]: *Dívidend* gives the Ráte into Príncipal and Tíme.
2 And the Tíme into Rate, more Yéar into ág, the *Diviſor*[b].
3 *Preſent Wórth* is the Príncipal léſs thę Rebáte; to be paid-down[c].

a *Rebate*, or *Diſcount* of mony is the allowance made by the creditor out of a ſum of mony, due to him at the end of ſome certain time, in conſideration of the prompt payment of the remainder by the debtor——That ſum, payd-down inſtead of the principal, due hereaſter, may be calld the *Preſent-worth:* in regard that, if it were put-out to intereſt for the time that the diſcount is computed; it would amount to the principal, due at the end of that time. V. Intereſt.

b *For example:* Sold goods for £ 795 11 2: to be paid at 2 3 months: that is, one half at 3 months; and the other half at 3 months after that. If all the mony be payd-down; what muſt be diſcounted? (Anſw.)

For the 1*ſt* 3 *months:*——for the *Dividend:* 6 *l.* (the rate) × 3 mo. (the time) = 18 × 397, 779 *l.* (half the mony to be paid-down) = 7160,022. Then——for the *Diviſor:* 3 months (the time) × 6 (the rate) = 18 + 1200 [the year; to-wit, 12 months (to correſpond with the ſpecies of time aforementiond) into ag = 1218 ——Then, 7160,022 (the dividend) ÷ 1218 (the diviſor) = 5,878; that is £ 5 17 6 3.

For the 6 *months:*——for the *Dividend:* 6 × 6 × 397, 779 = 14320044——for the *Diviſor:* 6 × 6 + 100 × 12 = 1236.———Then, 14320044 ÷ 123 = 11, 585 that is £ 11 11 8 2.

c For the *Preſent-worth:* 5,878 (the 3 month's rebate) + 11,585 (the 6 month's rebate) = 17,463 (the whole diſcount) which, deducted from 795.558 (the whole ſum) leaves 777,095 (that is £ 777 1 10 3) for prompt payment.

REDUCTION.

1 R'ED. By the númber of óne name, that makes óne of another,

2 *'Upwards* to Gréater DIVIDE: to Leſſer MÚLTIPLY *Downwards.*[a]

3 *Dífferent Námes* to the léaſt of the lóweſt redúce, add, and value.[b]

a To *Reduce*, or bring to a different denomination of equal value*,

1. *Upwards* (e. g.) 12014 farthings to pounds (a Greater name) Divide —— either by 960 (the number of farthings that make one pound) the quotient will be 12 (viz. pounds) the remainder, 494: which, multiplied into 20 (*s*, the number of the next inferior denomination that makes 1 pound) gives 9880. This, divided by 960 (the common diviſor throughout all the operation) gives = 10 *s:* the remainder 280. Then 280 × 12 (*d*, the number of the next denomination that makes 1 ſhilling) = 3360 ÷ 960 = 3 d: the remainder 480. Then 480 × 4 (*q*, the next name) = 1920 ÷ 960 = 2 q. So, the work will ſtand, as in the margin, n. I.——— or (proceeding by ſteps) divide by 4, to bring it to pence: that, by 12; to bring it to ſhillings: that, by 20 (or halving all but the laſt) to bring it to pounds. So the work will ſtand (moſt commodiouſly) as in the margin, n. II: in working of which, I ſay: 4 in 12: 3, which I ſet underneath (V. Diviſion, line 7). Then 4 in 0 (the next figure) 0 (to be ſet down). Then 4 in 1 (the next figure) 0 (to be ſet down

(I)

12014 |	960
2414	12 l.
. 49	
20⸸	
9880 |	10 s.
. 28	
12⸸	
3360 |	3 d.
. 480	
4⸸	
1920 |	2 q.

(II)

q.	12014	*— 2†
d.	3003	— 3
s.	250	— 10
l.	12	———

* Divided by 4 *q*, gives the quotient as underneath, &c.

† Remainders, plac'd againſt their proper denominations.

⸸ NB. The *multiplicators* need not be ſet-down: nor the *diviſors*, within the compaſs of the multiplication-table. V. n. II.

down alſo. Then 4 in 14 [the next figure, with 1 (the preceding remainder) prefixt] 3, &c. ——— (NB) *In different denominations* the method is the ſame: Thus, The work, in the margin, may be wrought as follows: 3 in 11: 3, and 2 over; which being prefixt to the next figure (in the value of its place) will make 22. Then, 3 in 22: 7, and 1 over; which prefixt to the next figure (viz. 12 s) and reduc'd to the ſame name, will make 32 s. Then, 3 in 32: 10, and 2 over; which reduc'd to the next name (viz. pence) will be 24; and 7, added, makes 31. Then, 3 in 31: 10, and 1 over; which (in the next name) is 4 q; to which the 2 q, being added, will make 6. Then 3 in 6: 2. So that the work is finiſht. V. Practiſe, note [c].

	£ 112	12	7	2
3)	37	10	10	2

2. *Downwards* (e. g.) £ 12 10 3 2 to farthings (a Leſſer name) Multiply 12 *l* into 20 (the number of *s* in a *l*) taking in the 10 *s* (viz. adding units to units; tens, to tens). Then, multiply the ſhillings into 12 (the number of pence in a ſhilling) to bring them to pence; adding the 3 *d:* and ſo on, to the loweſt denomination: The work will ſtand as in the margin. —— NB. *Remember always* to mark the denominations, as you advance; to prevent confuſion and miſtakes; which, without ſuch caution, will be apt to creep into the operation.

l.	. . . 12	10	3	2
	20			
s.	250 . . +	10		
	12			
d.	3003 . . +	3		
	4			
q.	12014 . . +	2		

To divide the value of any ſpecies into different denominations, the number of which ſhall be equal; Reduce the denominations to the loweſt name; by which divide the ſpecies, reduc'd to the ſame name.——*Exa.* In a moidore, how many nobles, crowns, ſhillings, groats, 3-pences, 2-pences and pence; of each an equal number?——*Anſw.* 2 of each. For, 6 *s* 8 *d* (a noble) + 5 *s* (a crown) + 1 *s* + 4 *d* + 3 *d* + 2 *d* + 1 *d* = 162 *d*. Then 324 *d* (a moidore) ÷ 162 *d*, the (ſum of the particulars, in the loweſt name) = 2.

* *A Diagonal Circular* INSTRUMENT, conſiſting of 7 concentrical circles (of mony, weights, and meaſures) beſide the two outermoſt, which are lines of numbers; was contrivd, and publiſht in 1721, by Mr Hatton, for reduction of mony, weights, and meaſures into decimals; or for finding (with eaſe and ſpeed) the value of thoſe fractions, and for multiplication, diviſion, and evolution. ——But, as all inſtrumental arithmetic is of little uſe, except to thoſe who are unacquainted with the doctrin of numbers; which is the buſineſs of this treatiſe: it may ſuffice juſt to have mentiond it. Thoſe who have a mind to ſee an account of it, will find it in his ſyſtem of arithmetic, p. 282.

b *Different names* (e. g.) 4 guineas, 3 half-guineas, 2 nobles, 1 livre, To reduce to the value in Engliſh mony ——The leaſt name of the loweſt particular being pence, all the particulars are to be reduc'd to pence, as afore directed (note [a] 2). Then, being added-together, the ſum total is to be valued, or reduc'd to the higheſt name of our mony, viz. pounds.——Thus

			s	*d*			*d*			
4 guineas	*gradually reduc'd* will be		84	-	*that is, in the loweſt name*		1008	*which being reduc'd* (by n. [a] 1) *give*		as, here, underneath.
3 half guineas			31	6			378			
2 nobles			13	4			160			
1 livre			1	6			18			
		s.	130	4	=	*d.*	1564	—	4	
						s.	130	—	10	
						l.	6	—		

RULE OF THREE.

1 RULE-OF-THREÉ[a] to perfórm; State, Prepare, Work, ánd Value . . quote, reſt.[b]
2 *Sta*] *Quéſtion*-term, LÁST, correſpónds to the FÍRST: to the MÍDDLE, the *Anſwer.*
3 *Pre*] DÍFFERENT námes to the *Leaſt* redúce; to the *Sáme*, CORRESPONDING.

Wor]

4 *Wor*] Láſt into Mi'ddle by Fírſt: the *Reverſe*;
if Móre, leſs; or Leſs, more.

5 *Val*] Quótient redúce: the Remáinder's
the nūm. of denóm. the diviſor.[c]

6 Dúplicate, by rátios of *Squares*[d]; and
Tríplicate, by rátios of *Cubes*[e], work.

7 Sesquiplica'te, by the rátios of *ſquáres to
cubes*; ánd vice verſa[f]:

a *NB*. Under this head of Proportion (in regard to the name of the rule*) I have given the doctrin of the *ſingle* rule; or the method of finding a 4th proportional to 3 numbers given †: the *double* rule (or the method of finding, by two ſtatings, a 6th proportional to 5 numbers given) is better explaind under the article Golden rule; in a manner, that I do not find any-where elſe deliverd. V. Golden rule.

* *The Name* of the Rule-of-three imports but 3 terms in a queſtion propoſd to be ſolvd by it. But it may not be improper to caution the learner, that (NB) Queſtions, belonging to this rule, may be propoſd in ſuch manner as that— There ſhall be *a ſuperfluous term*; which, though it makes a circumſtance in the queſtion, yet is not concernd in the proportion; becauſe it is equally ſo in both the ſuppoſition, and demand. The ſuperfluous term is always known by being twice mentiond; either directly, or by ſome word that refers to it. For example: If 3 men ſpend 20*l* in 10 days; how much, at that rate, will they ſpend in 25 days? Here the 3 men, though not directly mentiond a ſecond time, being referd-to by the term 'they', appears to be a ſuperfluous term; the proportion being among the other 3 given terms, with the number ſought: ſo that any number of men may be as well ſuppoſd, as three.——— Sometimes there are *two ſuperfluous terms*; as in the following example. When wheat is at 12*s* per buſhel, the 6-penny loaf of bread is (by ſtatute) to weigh 1 *lb* 4 *oz* (troy-weight). What ought it, then, to weigh, when wheat is at 9*s* 6*d* the buſhel? Here are, plainly, two ſuperfluous terms; to-wit, 1 buſhel, and 6-pence. Theſe, therefore, being caſt-out, the ſtating will be as follows: As 12*s* to 1 *lb* 4 *oz*: So 9*s* 6*d* to 16 *oz* $14\frac{2}{57}$ *dw*.

† *A Caution* is neceſſary to be here given; leſt the learner be deceivd, and take thoſe things to be proportional, which are not ſo. E G —— Suppoſe a weight, moving by its own gravity, deſcends 20 feet in 2 ſeconds of time; and it be requird to find how many feet it will fall in 10 ſeconds? The ſtating will ſtand thus: 2 : 20 : : 10: and 100 would be the anſwer. But, this number does by no means anſwer the queſtion: becauſe a falling weight does not proceed with an equable motion, but with an accelerated one; its motion being ſo much the ſwifter, by how much the longer it is in motion.——Again, If a veſſel, that will hold (ſuppoſe) 12 gallons, be filld with water, and a hole be made at the bottom; and, through it, there runs-out 3 gallons in 2 minutes of time; and it be demanded in what time all the water will run-out. According to the tenor of the rule of three, it will be anſwer'd in 8 minutes. And, indeed, this would be the true anſwer, ſhould the water continue to run-out with the ſame velocity, with which it began to run. But it is quite otherwiſe: for the celerity of the water runing-out continually decreaſes: and, by how much the leſs remains of water to run-out, by ſo much the ſlower it runs-out; that is, It runs-out with a greater velocity when it is near full (for inſtance) than it does, when it is but half-full: becauſe the inferior water runing out, is (in the former caſe) preſt by a greater weight of the ſuperior water, than it is in the latter.

b This is the *whole work* of this excellent rule in miniature: and a little attention hereto (as explaind in the 4 following verſes) will enable a learner to diſembroil the perplexity of the moſt complex queſtions——*More particular directions*, for the application of this rule, in all the variety of circumſtances where proportions ariſe, *it is*, as Mr Malcolm (arithm. p. 536) obſerves, *impoſſible to give.* For, Queſtions may be leſs, or more complex; comprehending various queſtions of proportion connected in their circumſtances; either to bring-out ſeveral numbers requird, or as ſo many neceſſary ſteps towards the finding of one number requird: and, beſides the proportions contained in a queſtion, there may be other operations of addition and ſubtraction, ſimple multiplication or diviſion, neceſſary either to make-out the terms of a proportion; or, after the proportions are ſolvd, to find-out ſome numbers ſought:

or a number to be further applied towards finding numbers ſought: in ſhort, to ſatisfy ſome condition of the queſtion in the progreſs of the work. The managing of ſuch queſtions depends upon the arithmetician's judgment in diſtinguiſhing all the parts of the queſtion; and knowing what each requires according to the true ſenſe and import of it; and of the ſeveral operations of arithmetic, and particularly of proportion: of all which he muſt have a clear and ready idea. And, as there is no other general direction, that can reach all caſes; the only thing more, that can be done to help one to acquire the neceſſary capacity for all uſeful queſtions, is to make the application particularly to ſuch variety, in all the common ſubjects and branches of buſineſs; that he, who underſtands theſe, may be ſupposd capable to do any other of the ſame, or any other uſeful kind. To this purpoſe are all the other common rules that are generally brought-in after the rule of three; and are but applications of it: a full account of which is to be found under the proper heads. And I have added (in the latter part) for exerciſing herein (with a hint, where neceſſary, for the ſolution of them) a liſt of complex queſtions, which come not ſo well under any of thoſe titles, that generally make-up the ſubſequent part of the ſyſtem, in common arithmetics.——*In caſe of Fractions,* you reduce the extremes to one denominator; and, that being neglected, you work by the numerators only: ſince that denominator would be a multiplier both in the numerator and denominator of the quote, as it comes out firſt in fractional form: and therefore, both being divided by it (or, which is the ſame, neglect it in the operation) the quote will ſtill be the ſame. So, to multiply by $\frac{5}{8}$, and divide the product by $\frac{3}{8}$, is the ſame as multiplying by 5, and dividing by 3. And, thus, with regard to the rule of three (1) If $\frac{3}{8}$ of a yard coſt 8 *s* what will $\frac{5}{8}$ of a yard coſt? The anſwer will be the ſame as in this ſtating 3 : 8 : : 5. (2) If 4 $\frac{3}{5}$ ounces coſt 14 *s*; what will be the price of 9 *lb*, or 2304 *oz*. For anſwer, The firſt reducd is $\frac{23}{5}$, and you may turn either 14 or 2304 into the form of a fraction, whoſe denominator is 5; and then work with the numerator only: and ſo it will be either 23 : 70 : : 2304; or 23 : 14 : : 11520. The anſwer is 7012 $\frac{4}{23}$.

To Exemplify: What will the carriage of cw 17 3 11 come-to, at the rate of 7 *s* 6 *d* per cw?——*Anſw.* Here, as in all other queſtions in this rule, there are 3 terms: one of which an inquiry is made-about, on the proportion ſpecified by the other two: Then,

I. *To State:* Place —— cw 17 3 11 (the term, which moves the queſtion; or, about which the inquiry is made) Laſt —— cw 1 (the term, which correſponds to it, as being of the ſame name, or nature) Firſt —— *s* 7 6 (the other term) in the Middle [1]. This done,

II. *To Prepare:* Reduce —— the Different names of the Middle term (viz. 7 *s* 6 *d*) to the *leaſt*: And thoſe of —— the Correſponding terms (viz. cw 1, and cw 17 3 11) to the *ſame*: viz. by reducing the greater to the leſſer; that is cw 1 to 112 *lb*; that being the leaſt name of the other term, when reduc'd[2]. Then,

III. *To Operate:* ——In *direct* proportion[3] (1) Multiplie 1999 (the laſt term) into 90 (the middle term) And (2) Divide the product (viz. 179910) by 112 (the firſt term)[4] ...The QUOTIENT (1606) will be the Anſwer (viz. 1606 *d*; as correſponding to the middle term, which is *d.*)... The REMAINDER will be the numerator of the fraction of the anſwer; and the diviſor, the denominator (viz. $\frac{38}{112}$, or 38 parts of a penny divided into 112) —— In *reciprocal* proportion[5] (1) Multiply the firſt into the middle term; and (2) Divide by the laſt ... The Quotient and Remainder will be, as afore.

IV. *To Value* —— the Quotient; reduce the name to the higheſt denomination; as is taught in the article Reduction. To value —— the Remainder; bring it to the next Lower name, as is taught in the article Fractions.

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[1] *So*, the work —— will Stand thus: cw 1 : 7 *s* 6 *d* :: cw 17 3 11 —— to be Read thus (1) As 1 to 7 6 : So 17 3 11 to &c. (2) Or (accommodated) thus: If the carriage of 1 *cw* coſt 7 *s* 6 *d*; What will that of *cw* 17 3 11 come-to?

[2] *So*, the work will Stand thus: 112 *lb* : 90 *d* :: 1999 *lb*.

[3] When More (for inſtance, weight, as in the caſe) requires More (for inſtance, mony-to-be-paid, as here): or, contrariwiſe, Leſs requires Leſs; the queſtion is ſaid to be in *direct* proportion, or the rule-of-three direct. In this caſe the rule for operation is as follows, n. III.

[4] *If the dividend be Leſs than the diviſor*; reduce it to a lower denomination, that ſhall (at leaſt) be equal to the diviſor.——*For example:* What are 5 yards of ribon

worth; $63\frac{1}{2}$ yards of which coſt 5 *l*? Here 20 (*qrs*, the laſt term) × 5 (*l*, the middle term) = 100: which, being leſs than the firſt term reduc'd to quarters, to-wit 254, by which it ſhould be divided; I bring the 5 *l* to *s*, to-wit 100; and, having multiplied it into 20 (the laſt term) I find it comes to 2000: which, being divided by 254, gives 7 *s* for the anſwer; with a remainder, to be valued according to the directions, n. IV.

5 When More requires Leſs; or Leſs, More: the queſtion is ſaid to be in *reciprocal* proportion, or the rule-of-three *inverſe*; or, according to the ancients, the *backer* rule. In this caſe, the Reverſe of the rule takes place——For example, If a penny-white-loaf ought to weigh 8 *oz*, troy-weight, when wheat is ſold for 6 *s* the buſhel: What ſhould it weigh, when wheat is ſold for 4 *s* the buſhel? (In anſwer) The queſtion being ſtated [6 *s* : 8 *oz* : : 4 *s*] it will appear (on conſideration) that Leſs (viz. price of wheat) will require More (viz. weight of bread). The rule of operation, therefore, is the reverſe of the other, viz. 'Firſt into Middle by Laſt': which will ſhow that the penny-white-loaf, when wheat is ſold for 4 *s* the buſhel, ought to weigh 12 *oz*.

d *For example:*——in Direct proportion (1) What is the area, or ſuperficial content of a circle, whoſe diameter is 8; when the diameter of a circle being 2, the area is 3.1416, Anſw. 50.2656. for, As 4 (the ſquare of the diameter of the circle given) to 3. 1416 (the area): So is 64 (the ſquare of the diameter of the circle ſought) to 50.2656. (2) What is the diameter of a circle, whoſe area is 50.2656? ſuppoſing the diameter of a circle, whoſe area is 3.1416, be 2, as it really is. Anſw. 8. for, as 3.1416 (the area of a circle) is to 4 (the ſquare of that circle's diameter): So 50.2656 (the area of any other circle) to 64, the ſquare of the diameter of that circle; the ſquare root of which is 8.——in Reverſe proportion (1) Suppoſe the length of the pendulum of our common clocks to be 39 inches, as it is very little more. We know that ſuch clocks vibrate ſeconds, or 60 times in a minute. What, then, will be the length of a pendulum that vibrates half-ſeconds, or 120 times in a minute? Anſw. $9\frac{3}{4}$ inches. for, As 3600 (the ſquare of 60 vibrations) to 39 (inches of pendulum): So are 14400 (the ſquare of 120 vibrations, which are More than 60*) to $9\frac{3}{4}$. (2) Suppoſing a pendulum of 39 inches vibrates 60 times in a minute; How many times does that pendulum vibrate in a minute, whoſe length is 9 ¾ inches? Anſw. 120. for, As 39 (inches) to 3600 (the ſquare of 60 vibrations) So is 9 ¾ (inches; which are Fewer†) to 14400 vibrations; of which the ſquare-root is 120.

* And, therefore, require fewer inches of the pendulum: for, the longer the pendulum, the fewer the vibrations in
† a minute; † and the contrary.

e *For example* —— in Weight (1) If a ſphere, or ball, which is 8 inches diameter, weigh 48 *lb*; what will another ſphere of the ſame ſpecies of matter weigh, whoſe diameter is 4 inches? Anſw. 6 *lb*. for, As 512 (the cube of the diameter 8) is to 48 *lb*: So is 64 (the cube of the diameter 4) to 6 *lb*, the weight requird. (2) If a ſphere, weighing 48 *lb*, be 8 inches in diameter; What is the diameter of another ſphere of the ſame matter, whoſe weight is 6 *lb*? Anſw. 4. for, As 48 *lb* is to 512 (the cube of the diameter 8): So is 6 *lb* to 64 (the cube of the diameter ſought) whoſe root is 4.———in Solidity. (1) Suppoſe the ſolid content of a ſphere be 4.1888, whoſe diameter is 2: What is the ſolidity of another ſphere, whoſe diameter is 4? Anſw. 33.5104. for, As 8 (the cube of the diameter 2) to 4.1888 (the ſolidity of the ſphere given): So is 64 (the cube of the diameter 4) to 33.5104, the ſolid content of the ſphere ſought. (2) The diameter of a ſphere being 2, whoſe ſolidity is 4.1888, what is the diameter of another ſphere, whoſe ſolidity is 33.5104? Anſw. 4, the cube-root of 64. for, as 4.1888 : 8 : : 33.5104 : 64.

f *For example:* The earth finiſhes its period in 365 ¼ days; and its diſtance from the ſun is 86,000,000 miles. What, then, is the diſtance of mars from the ſun, ſuppoſing its period to be 625 days? Anſw. 130,000,000 miles. for, As the ſquare of the time, in which any planet finiſhes its periodical revolution, is to the cube of its diſtance from the ſun: So is the ſquare of any other planet's time, to the cube of its diſtance.

14 NO 63

SUBTRACTION.

BRITISH MUSEUM

1 SÚB. (1) Place the -tractor únder -trahénd,
as in adding (2) Then take
2 'Units from únits, and ſo on (3) Write the
Difference, the Réſt or Remainder[a].
3 *Nóte* (I) When from Léſs: take from óne of
the néxt in that náme; add the leſs; pay[b]:
4 And (II) When from Séveral, óne; or Re-
vérſe: make them óne by addition[c].

a *For the Manner of operation*, having plac'd the ſubtrahend and ſubtractor,* as in the margin (according to the inſtructions given under the article Addition)——— The *common* way is this: Begining at the right hand, or units; Say, 2 from 6, and there remains 4. Then, 3 from 5, &c.———More *commodiouſly*, addition-wiſe, thus: 2 and (and as much as will make 6, viz.) 4 (which ſet down as you pronounce it) is 6. Then, 3 and 2 is 5 &c.

Subtrahend . . .	4256
Subtractor . . .	2132
Remainder . . .	2124

* Thus, in ſubmiſſion to Mr Malcolm's authority, and by way of alluſion to the terms of dividend and diviſor; I am content to call the factors of this operation: though, in propriety, the Subtrahend is the number to be ſubtracted; and that, from which it is to be ſubtracted, ſhould be calld the Compound number; as being compoſd of the leſſer number, and the difference ——Some, with more propriety, call them the *minorand*, and the *ſubducend*.

b *For example:* To operate the examples in the margin:

In the Iſt—Say: 2 from 1, I cannot: but 2 from (one of the next row, in the name of this, viz. units; that is, from) 10: there remains 8; and 1 (the leſs number) is 9. Then (going-on to the next ſtep, ſay) 1 (that I borrow'd) and 4 is 5: 5 from 2 I cannot; but &c. as afore —— Or rather (addition-wiſe) ſay: 2 and (as much as will make the amount to the next row, viz. 10, with the 1 added, viz. 11) 9 (ſeting it down, as you pronounce it) is 11. Then 11, being the laſt found, admoniſhes what is to be paid in proceeding. So, looking on 4 (the next ſtep) ſay (paying the 1, you carry) 5, and (ſo much as will make 10 and 2, that is 12; viz.) 7, is 12. Then 7 and 7 is 14; &c. *NB.* This method is vaſtly advantageous in the operating of diviſion. V. Diviſion.

I	From . .	12365421
	Take . . .	3578642
	Reſts . . .	8786779

II	S^d.	12 *l.*	5 *s.*	6 *d.*	2 *q.*
	S^r.	6	3	7	3
	R^r.	6	2	3	3

In the IId —— Say: 3 *q* from 2 *q*, I cannot: but 3 *q* from (1 of the next row, viz. 1 *d*, that is) 4 *q*; and there remains 1 *q*; and 2 *q* (viz. the leſs, from which 3 could not be ſubtracted without borrowing) is 3 *q*; which ſet-down; and, proceeding to the next row, ſay 1 (that I borrowd) and 6 is 7: 7 from (1 of the next row, viz. 1 *s*; that is) 12 *d*; there remains 5, and 6 is 11. And ſo on. (*NB*) 'I cannot' 'and there remains' &c. after ſome practiſe, may be omited; as rather embaraſſing, than expediting or explaining the operation. Thus, the laſt explication will run thus: $3 - 4 = 1 + 2 = 3$. Then $7 - 12 = 5 + 6 = 11$. And ſo on.——The other method (addition-wiſe) is not, here, ſo commodious.

c *For example:* Let it be propoſd to ſubtract 560 from 467 + 235; or 345 + 432 from 978: the work of the former will ſtand, as in the margin, n. I; of the latter, as in n. II. —— And the ſame is to be done,

I		467
		235
	Sub^d.	702
	Sub^r.	560
	Rem^r.	142

II	Sub^d.	978
		345
		432
	Sub^r.	777
	Rem^r.	201

when two or more ſeverals are to be ſubtracted from two or more ſeverals.

TABLES*

OF

Coins, Measures, Weights:

I. Britiſh.

1 *How many This make That:* From thís to
that múltiply thĕ numbers[a].

2 *Míxt numbers* Bríng to the Léaſt name: To
válue 'em, Dĭvíde by the ſame name[b].

Coins & Mony.

3 Farthing - o . pén - be . ſhil - ez . *Pound* . :

Measures.

4 Ale, Beer:[c] gál-k, re (beer-ou: ale-k) fir-d.
kil-d. bár-a,re . ho-d . *Butt*.

5 Cí'rcular: mínutes (with múltiples[d]) -auz.
deg-ïy . ſign-ad . *Circ*.

6 Cloth: inch-d,ró . nail-o . Quar-Yard =
Ells: eng-ú . flem-i . french-au.

7 Dry': gal-e . péck-o . buſh-e . ſtrike-e .
coomb-é . quar-u . wey-d . *Laſt*.

8 Liquid: gíll-o . mutch-é . chop-e . pint-e .
quárt-o . gal-as . *Hog*.[e]—V. Ale, Wine.

9 Lóng: line-be . ínch-be . foot-i . yar-d .
fáth-e,tro . r-óz . fur-ei . m-i . *League*.

10 Númb:[f] dicker-az . kip-üy . quire-el . ream-
ǘg . timber-öy.

11 Quántity:[g] *Róll* = dozen-u Skins . . ȅlls-
bazy M, 'Oz . . hun-ro . Barb-tob.

12 Squáre : in-afo . foo-n . yár-ty,ro . p-oz . rood-ó . acre-ſoz . *Mile*.

13 Tímber : inch-boídei . foot-ép . *Yard* . . . Foot-óz round, foot-úl hewn : a *Load*, *Tun*.

14 Tíme : ſecond-áuz . minute-auz . hour-ef . day-tául, u, fei, up[h] . *Year*.

15 Wíne : gal-ak . rún-a,tro . b-á,rt . ti-b,re . hog-a,rt . púnch-b,re . pipe-e . *Tun*.

16 *Gallons*[i] = bár-ta,re . tierce-od . hóg-ſi . pun-eíf . pipe-adau . T-eld.

Weights.

17 A′verdupois [a]: dram-as . ounce-as . pound-ábe . hun-ez . *Tun*.

18 Troy . . *Gold:* [b] grain-ef . pénw-ez . Ounce . . *Pot :* gr-ez . ſcr-i . dr-ei . oun-ad . *Pound*.

a *Wool :* pound-oí . clove-e . ſtóne-e . tod-au, re . wey′-e . ſack-ad . *Laſt*.

b *Mint-monyers :* blank-éf . periot-ez . droíte-do . mite-ez. *Grain*.

Proportions & Diſtinctions.

21 Foot : *Britiſh*, áth . *Greek* azyp . *Rome* (coſs) naup (st) oúpe (ves) oukau.

22 *Par*, azáuk . *Rhin*, azít . *Venice*, abſe [k]. *Ams*, nod . *Cópenhag*, nául.

23 Gállon *cube-ínches* contaíns . . *Dry :* eſeí, fru . . *Beer :* éke . . *Wine :* eta[l].

24 Grains . . *Engliſh*-bif, re make . , *French*-alei . . *Dutch*-apou.

25 Oúnce *Aver* Líghter than *tróy* (near a 12th) as úa to uau.

Póund

26 Póund *Aver* Heavier than *tróy* (near a 6th) as áoi to ao . . . Equal, in troy-weight, to ounce-ſo, pén-ba, grain-al,re [m].

29 Stone = *Pounds:* Beéf-k (heref-ad, pembak, north-ás) Glaſs-u . Wax-ei . . . Woól: her-be, gloce-bu . Spíce cin nu pép ſugar-at, re. The *reſt* af [n].

Observations on ſundry Particulars[*].

A Abbreviatures: Barrel. Buſhel. Carat. Chópin. Circle. Degree. Dram. Fathom. Firkin. Foot. Furlong. Gallon. Grain. Hogs-head. Hundred-weight. Inch. Kilderkin. Line. Mile. Mutchkin. Ounce. Penny. Penny-weight. Perch. Punchion. Quarter. Rod. Runlet. Scruple. Shilling. Tierce. Tun. Yard.

E Equivalents: Broad-piece, carolus, jacobus. Butt (of malmſey, ſack, &c.) pipe. Carnock, Cornock, coomb. Digit, inch. Laureat, carolus. Lundreſs, ſilver-penny. Perch, rod. Pieces, guineas. Pole, rod. Prime, minute. Quintal, hundred-weight. Seam, quarter. Sterling, penny, penny-weight. Tertian, tierce. Tun, wey. Twelve-month, year.

O Omitted —— *Coins:* Achiſon, .20 *q*. Angel 10 *s*. Angelet 4 *s*. Bawbee, .15 *q*. Bezant, 3 *l*. 15 *s*. Bodle, .05 *q*. Cardecue, 19.50 *d*. Carolus, 23 *s*. Croſs-dagger, 11 *s*. Crown, 5 *s*. Florence, 6 *s*. Guinea, 21 *s*. Groat, 4 *d*. Harper, 9 *d*. Jacobus, 25 *s*. Maile, $\frac{1}{2}$ *q*. Mark 13 *s*. 4 *d*. Mony, 4 *s*. 6 *d*. Noble 6 *s*. 8 *d*. Plack, 10 *q*. Rial 10 *s*. Salute, 6.96 *s*. Sexling, 15 *s*. Sovereign, 22.50 *s*. Teſtoon, 18.84 *d*. Thryling, .07 *q*. Unicorn, 6 *s*. Unite, 22 *s*. Urchin, 12 *q*. ——*Meaſures:* Barley-corn, $\frac{1}{3}$ of an inch. Cubit, $\frac{1}{2}$ a yard. Firlot, 31 pints. Fortnight, 2 weeks. Gill, $\frac{1}{4}$ of a pint. Hair's-breadth, $\frac{1}{48}$ of an inch. Hand, 4 inches. Month, 4 weeks, or $\frac{1}{12}$ of a year. Pace, 5 foot. Palm, 3 inches. Pottle, 2 quarts. Scruple, $\frac{1}{1680}$th of an hour. Span, 9 inches. Week, 7 days.——*Weights:* Clove, 8 *lb*. Pack, 120 pounds. Pig, 21$\frac{1}{2}$ ſtone. Prime, $\frac{1}{24}$ of a grain.

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[*] The *ancient* coins, weights, and meaſures; and the proportion they bear to each other; may be learnt in my

Mnemonics: where, by getting a few lines, a child may treaſure up (in his head) all the large tables of Dr. Arbuthnot. V. Pref. paragr. 6.

a *For example:* How-many farthings make a pound? —V. line, 3. (Anſw.) $4 \times 12 = 48 \times 20 = 960$.

b *For example:* How many gallons of wine in a barrel? —V. line, 15. (Anſw.) $18 \times 1\frac{3}{4} = 31\frac{1}{2}$ (V. Fractions, l. 17) Or —— Bring $1\frac{3}{4}$ to the leaſt name, viz. quarters, thus: 4×1 (the integer) $= 4 + 3$ (quarters) $= 7$ ——Then $18 \times 7 = 126$ —— Then $126 \div 4$ (the leaſt name) $= 31\frac{2}{4}$ or $\frac{1}{2}$. V. note [1].

c *That is:* $8\frac{1}{2}$ gallons [but (about London) 9 of beer, and 8 of ale] make a *firkin* : . 2 firkins make a *kilderkin* . . 2 kilderkins, a *barrel* . . . And ſo in the reſt.

d *That is:* The ſubdiviſions of minutes are ſeconds, 60′; thirds, 60‴; &c.

e *In Scotland* theſe are their common denominations of liquid meaſure —— But it is to be obſervd that (1) A ſcotch mutchkin is almoſt an engliſh pint; and that (2) The exciſe in Scotland, ſince the union of the two nations, is calculated upon engliſh meaſure.

f *Collective names of Numbers* (for goods accounted by the *Tale*) are—Such as are us'd inGeneral: as *Brace, Couple*, 2: *Dozen*, 12: *Groſs*, 144 (i. e. 12 dozen) Great-*groſs*, 1728 (i. e. 144 × 12): *Leaſh*, 3: *Score*, 20. —— Such as are us'd in ſpeaking of Particular things: *Dicker:* of hides, knives, iron-bars, bundles of necklaces. *Kip:* of goat-ſkins. *Quire, Ream:* of paper. *Timber:* of ſkins of fitches, grays, genits, martens, minks, ſables.

g *Collective names of Quantities* are what frequently occur; and which to be quite ignorant-of is a little unſatisfactory —— I have, therefore, here, given a ſpecimen of them; as being the proper head, under which they were to be conſiderd: and, but one*; becauſe an alphabetical liſt of 'em in proſe will be full as much as any one would wiſh-for. V. Quantities.

* ROLL = *Dozen:* 5, of Skins of parchment —— *Ells:* 1100, of Minſters and Ozenbrigs. —— *Quintals:* $\frac{1}{4}$, of barbadoes-Tobacco.

h *That is* (taking the denominations backward) 365 days, 5 hours, 48 minutes, 57 ſeconds, &c. ——NB. (1) In the royal Navy, the men are payd their wages by the following table: a Year = 13 months, or 52 weeks, or

364

364 days: a Month = 4 weeks, or 28 days. (2) For ſums, where no great exactneſs is requir'd, they ſay: a Year = 12 months, or 365 days: a Month = 30 days. (3) But to be more exact, the Year ſhould be reckond $365\frac{1}{4}$ days. V. Time, in note [o].

i Multiplication of fractions in *Wine-meaſure* being ſomewhat operoſe (V. note [b]) I have here given the contents of each in gallons; which, by an eaſy diviſion, will give the contents of any, in any. For example: How-many tierce in a tun? Anſ. 252 ÷ 42 = 6.

k *That is:* The Britiſh *foot* being divided into 1000 parts: thoſe of the following places are, in proportion thereto, as in the rule: to-wit, the old Grecian foot 1007: the Roman, as found on the monument of Coſſutius, on that of Statilius, and on a congius of Veſpaſian; as ſpecified by the numeral letters ——— NB. Hereby, alſo, may be found the proportion of the engliſh *inch* to thoſe of the other places here mentiond, thus: As 1000 (the ſuppoſd parts of an engliſh foot) to 1068 (for the french foot) So 12 (the number of inches in an engliſh foot) to 12.816 (for the length of the french foot) i. e. 12 inches, and ſomething more than $\frac{3}{4}$. And, conſequently, there being 12 lines in an inch, the french inch is $\frac{3}{4}$ of a line longer than the engliſh.

l Theſe are the contents, according as they are now computed, and practiſd in the art of gauging —— But (1) The *Wine-gallon*, kept ſeald at Guildhall in London, according to act-of-parliament (11 H. 7. c. 4.) by which all wines, brandies, ſtrong-waters, mead, perry, cider, vinegar, oil, and hony are meaſurd; was found, by a nice experiment (in 1688) to contain but 224 cubic-inches. However, for ſeveral reaſons, it was, at that time, found convenient to continue the former ſuppoſd content of 231; and that all computations in gauging ſhould be made from thence. V. Wybard, tectometr. p. 289. (2) The common receivd content of the *Corn-gallon* (before the year 1697; when the legal wincheſter-buſhel was ſettled by an act of parliament, according to a ſtandard in his majeſty's exchequer) was $272\frac{1}{4}$ cubic-inches; in a mean (as it were) between the wine-, and beer-gallon. V. Ward, arithm. p. 36.

m So Mr Ward (arithm. p. 33) tells us, he found it by a very nice experiment. V. note o.

n *That is:* A ſtone —— of Beef is 8 *lb.* [but in Herefordſhire and parts adjacent, 12: in Pembrokeſhire, &c. 18: in the Northern counties, 16] —— of Glaſs, 5 *lb.* &c. —— in Racing Hay, Iron, Shot, &c. 14 *lb.* (NB) What other differences there are, in this or other denominations, will be given, with more propriety, under the article Quantities.

o WEIGHTS and MEASURES. The original of all weights, uſd in England, is ſuppoſd to have been a corn of wheat, gatherd out of the middle of the ear, and well dried (V. Yard) 32 of which were to make 1 penny-weight; and 8 pound of them (or 61440 grains) were to fill 1 gallon of wine meaſure. V. 9 H 3. 51 H 3. 31 E 1. 12 H 7. —— And, by ſtatute (14 E 3) there was to be but *one weight*, throughout this realm; viz. troy: by which are (now) weighd jewels, gold, ſilver, corn, bread, and all liquors; and by which the proportion of gravity, which any two bodies have to each other, is uſually tried in philoſophical experiments. Averdupois, therefore, ſeems to have been introduc'd and ſettled by cuſtom, viz. from giving good weight to ſuch commodities as are uſually weighd by it; which are ſuch as are either very coarſe and droſſy, or very ſubject to waſte; as all kind of grocery-wares, and phyſical drugs; as alſo, baſer metals, and minerals; and fleſh, butter, cheeſe, ſoap, tallow, &c. to which it was thought convenient to allow a greater weight than what the law had provided. Mr Ward (introd. to mathem. p. 33) by a very nice experiment, found that one pound averdupois is equal to 14 *oz.* 11 *dw.* 15$\frac{1}{2}$ *gr.* troy. V. Proportion of weights, art. Tables, l. 27. —— By the ſame ſtatute it was alſo orderd that there ſhould be but *one liquid meaſure* throughout the kingdom. Yet cuſtom has prevaild; and, there having been introduc'd a new weight (viz. the averdupois) we have, now, a ſecond ſtandard-gallon (viz. the wincheſter, for ale and beer) adjuſted thereto; and therefore exceeding the former, in proportion of the averdupois to the troy-weight.

BUSHEL of *Salt*, † and *Sea-*

Sea-coal, is 5 ſtricken, or 4 heapt pecks. 11 H. 7.4
—— † Salt (formerly) uſd to be bought and ſold by meaſure, as corn now is. But it is (now) ſold, from the pits, only by weight; reckoning 7*lb* averdupois to a gallon; 56 *lb*, to the buſhel; and 42 buſhels, to the tun; for freight: and 5 buſhels is one ſack; and 4 cw is 1 quarter.

CARAT. By *jewellers* the ounce is divided into 192 parts, calld carats: which are divided into grains, or $\frac{1}{4}$ and $\frac{1}{8}$ $\frac{1}{16}$ $\frac{1}{32}$ $\frac{1}{64}$ &c. parts.—— NB. (1) The *Carat*-FINE is the 24th part of the goodneſs of a piece of gold. (2) The *Carat*-PRICE is the 24th part of the value of a piece of gold: as, if the piece be 384, the carat-price is 16 pound. (3) The *Carat*-WEIGHT is the 24th part of the weight of the piece, or 192 grains.

CORN-*Meaſure* (ſcotch) are Peck-o, Búſhel-o, Boll-as, Chaldron —— But they are different meaſures from the engliſh of the ſame name.

CUBIC *Meaſure* is eaſily found from the ſquare, by multiplying the ſquare into its root. Thus 144 (ſquare inches) × 12 (the root of 144) = 1728 .. 9 (ſquare feet) × 3 (the root of 9) = 27.

FIRKIN = *pounds*: 56, of butter: 60, of ſoap.

FOOT (in the foreſt of Sherwood) is 18 inches.

GALLON —— of Ale and Beer ought to hold 10 *lb*, 3.426 *oz* of pure, or rain-water. —— of Wine, 8 *lb*, 1 *oz*, 11 *dr*.

GRAINS, uſd in weighing *diamonds*, are ſomewhat lighter than thoſe uſd in gold, &c.

HOGSHEAD. The *diſtillers* weigh their veſſels, when full: and, for a hogſhead allow cw 4 2 22, caſk and liquor. —— Of virginia-*tobacco*, a hogſhead contains from 2 to 5 cw, and upward.

NOBLE. Half-nobles were calld *half-pennies* of gold: Quarter-nobles, *farthings* of gold.

PECK. The legal wincheſter peck, according to the ſtandard-buſhel in the exchequer, contains 2 gallons: But, beſide this, there are *local* pecks; containing, ſome more, ſome leſs. The —— Lancaſter peck is 6 gallons.

PENNY-WEIGHT (ſcots) are grains-ep, drops-noid, oúnces-alule.

PIPE is computed to weigh about cw 9 2 17. V. Hogſhead.

Pound. From Perſia and Turkey (1) *raw-ſilk* is ſold by the averdupois pound: but the pound is accounted 24 ounces. (2) but *ferret*, *filoſella*, *ſleeve ſilk*, *&c.* by the common pound of 16 *oz.* —— To Reduce (1) *Great pounds* to Common; multiply by 3, and divide by 2. (2) *Common pounds* to Great; multiply by 2, and divide by 3.

Rial = Shill. Hen-s, az†. Hen-k, baro. El, al. Jáme-b, ſpur-al, roſe-iz —— † i. e. The rial (in Henry, the 6th's reign) was 10 *s*.

Rod. By the cuſtom of ſeveral counties, there is a difference in this meaſure —— in *Herefordſhire*, a Perch (1) of Denſhird ground is 12 foot (2) of Ditching, 21 —— the *Scotch* perch is $18\frac{3}{4}$ —— in the foreſt of *Sherwood*, 25 —— in *Staffordſhire*, 24.

Runlet of *wine* and *oil* (by 1 R. 3. c. 13) was to hold $18\frac{1}{2}$ gallons.

Time —— In Aſtronomical calculations, Days ought to be made the greateſt denomination: and then any number of days may be again reduc'd to years, by dividing them by 365 days, 5 hours, 48 minutes, 57 ſeconds. —— In Compound-intereſt, it is beſt to make 365 days = 1 year; and uſe no denomination between year and day —— On Common occaſions, months and weeks make a convenient diviſion of time; though they cannot conveniently be mixt in accounts. V. note[h].

Tun is reckond to weigh about 1890 *lb* averd. 2016 *lb* troy. V. Hogſhead. —— In *oil*, by the cuſtom of London, 236 gallons (calld, by merchants, the civil-gauge) is ordinarily ſold for a tun: except whale-oil (or oil from Greenland) which has 252 gallons to the tun.

Wool is commonly bought by the *tod*: but when it is ſtapled, or ſorted; it is ſold by the *pack*.

Yard. It is highly probable, that the various meaſures of lengths were (at firſt) deduc'd, by the dictates of nature, from the different dimenſions of the parts of the human body, whoſe names they ſtill retain. Thus, from thence were taken the digit or finger's breadth; a hand; a ſpan; a foot; a pace; and ſeveral others. But, becauſe the dimenſion of the human body is various and uncertain; and the proportion (in meaſures) muſt neceſſarily be fixt and certain: hence every nation has pitcht upon ſome ſettled and determind extenſion, to uſe in their meaſures of lengths. Upon this conſideration our engliſh laws have determind the extent of the meaſure; which is kept (for a ſtandard-one) in Guildhall in London, calld the yard ſtandard. And, to this, all the other engliſh meaſures of lengths ought to obſerve their preſcribd proportions. Thus, An engliſh foot is a third part of the meaſure juſt mentiond. Thus, alſo, an inch is the 36th part of the ſame. Therefore, thoſe, who deduce the dimenſion of an engliſh inch from three grains of barley, are not altogether right: ſince our inch has not any reſpect (at leaſt any immediate one) to thoſe grains (or any other) but to the forementiond ſtandard-meaſure.

2. Foreign.

COINS and MONY*.

1. *Accounts are kept*

in **DENMARK**	*by*	*Kreuxer*	2.24
Rixdollar = q	216	*Pfenin*[b]	.28
Hor	54	Rixdollar	216
Schelling	2.25	Marc-lub	72
FRANCE		Schelling	4.50
Livre	72	Pfenin[c]	.37
Sol	3.60	*Dollar*	216
Denier	.30	*Kreuxer*[d]	2.40
GERMANY		Rixdollar	216
Florin	144	Groch	6
Kreuxer	2.40	Pfenin[e]	.50
Pfenin[a]	.30	*Rixdollar*	216
Rixdollar	216	*Florin*	120

* The reader will be pleaſd to obſerve that —— In the *firſt* table, the monies are all expreſt in the loweſt denomination of ours: ſo that it will anſwer all the purpoſes of the largeſt tables (1) the difference of any two terms being found by ſubtraction (2) and how many of any make one of another, by diviſion. —— In the *ſecond* table (1) the Gold coins are expreſt in ſhillings (3) the Silver, in pence: for the readier eſtimation of them in the head: (3) the Reſt, in farthings, for the greater accuracy ——In *all* of them, the fractions are decimals; becauſe they are full as eaſy to be conceivd as the vulgar, and more ready for calculation.

[a] In *Switzerland, and moſt of the chief cities.* [b] In *Baſil, Francfort, Nuremberg, Straſburg, &c.* [c] In *Berlin, Hamburg, &c.* [d] In *Augſburg, and Bolzamont.* [e] In *Leipſic, & Naumburg.*

Kreuxer	2
Hellert	25
IRELAND.	
Pound	720
Shilling	36
Penny	3
Farthing	.75
ITALY	
Ducat	240
Gros di banco[g]	10
Florin	156
Soldi [h]	26
Livre	28.80
Sol	1.44
Denier[i]	.12
Crown	240.28
Julio	24.03
Grain	3.01
Quatrine [k]	.60
Ducat	240
Tarin	48
Grain[l]	2.40
Ounce	600
Tarin	20
Carlin	10
Grain	1
Piccoli [m] (ſere)	.17
MUSCOVY	
Ducat	408
Ruble	204
Grif	20.40
Altin	6.12
NETHERLANDS	
Pundt	518.40
Schelling	25.92

Penning	2.16
Pundt	540
Schelling	27
Penning [n]	2.25
Florin	86.62
Stuyver	4.33
Penning [o]	2.16
POLAND	
Rixdollar	216
Groch	2.40
Pfenin	.12
PORTUGAL	
Milrea	360
Rea	.36
PRUSSIA	
Florin	72
Groch	2.40
Pfenin	.12
SCOTLAND	
Pound	80
Shilling	4
Penny	.33
SPAIN	
Real de vellon	13.6
Maravedi	.40
SWEDEN	
Rixdollar	216
Copper-dollar	36
Rouſtique	1.12
TURKEY	
Abouquel	49.50
Meidein	1.50
Aſpre	.50
Purſe	112*l* 10*s*
—— *of Gold*	6750*l*

[f] In *Zurich.* [g] In *Candia*, *Venice*, *&c.* [h] In *Geneva.* [i] In *Bergamo*, *Bologna*, *Genoa*, *Mantua*, *Milan*, *Modena*, *Novi*, *Placentia.* [k] In *Ancona*, *Ferrara*, *Rome.* [l] In *Naples.* [m] In *Malta, and Sicily.* [n] In *Flanders, &c.* [o] In *Holland, &c.*

The

2. *Coins their Value.*

GOLD		Dutgen	5.62	Blanca	.34
Albert = *s*	13	Ebrew	48	Blaphace	7.20
Caſtellano	6.16	Gluckſtadt	7.65	Brummer	3.60
Copec	1.75	Hongre	30.62	Carolus	3
Doblon	18.03	Izelotte	33	Cornado	.10
Ducatoon	39.17	Lis	18	Dollar	36
Hongre	9.25	Marc	6.75	Double	.60
Imperial	11.25	Monaco	52	Dreyer	.90
Joannes (port)	36	Obolus	26.67	Liard	.90
Lewidor	20.32	Ort	10.80	Maille	.15
Lis	10.50	Patagon	54	Marc	9
Manca	7.50	Petite Piece	3.60	Muſkofske	1.25
Moidore	27	Polpoltin	12.75	Negenmaniken	60
Piſtole	16.50	Poltin	25.50	Ochavo	.80
Ride	3.75	Ponti	5.28	Oortje	1.20
Rider	6.50	Real de plata	27.20	Ore	3.37
Rixgould	2	Rix-gould	13	Patac	.60
Sol	25	Rix-marc	21.50	Pite	.08
Zecchin	9.45	Rix-ort	27	Plappert	2.82
SILVER		Roup	4.75	Polchen	1.20
Abra = *d*	12.20	Scheſdale	36	Poluſke	2.50
Batze	4.75	Shakee	3.50	Quarto	1.60
Blanc	2.55	Vintain	7.28	Rappen	.47
Carline	6.25	BILLON		Stooter	4.19
Caroline	17.25	Blaze = *q.*	3.45	Swaar	.60
Chavelet	1.44	Cavalot	1.80	Vintain	6
Chriſtine	13.50	Pignatelle	.49	Vording	20.16
Copec	1.25	Ratze	3.45	Whitten	3.36
Croutac	5.29	Vintain	6	TIN, &c.	
Danſch	18	COPPER		V. *Annotations, at*	
Derlingue	14.25	Albus	2.78	*the end of theſe*	
Drittle	36	Alleuvre	.56	Tables.	
Ducatoon	56	Bayoco	2.42		

The following it may be convenient to know-by-heart, reduc'd to ſhillings & pence, *as follows:*

GOLD: Ducat (holl.) ou í. Lewidore: ez o. Moidore: ep.

Piſtole (ítal.) as au . John (pórtugal) ís . Zechin : ou au.

Silver : Crówn, Patagón, Piece-of-eight, Dollar, Rix-dollar : o au .

Crúſade : o ei,re . Dúcat : u o,re . Dúcatoon : o ei.

Flórin : a az,re . Lívre : a . Márc : e i . Réal : e i,ru .

3. *Countries their Coins.*

Denmark. *Silver* : Danſch, Ebrew, Gluckſtadt, Hor, Rix mark, Rix-ort, Scheſdale.

France. *Gold* : Crown, Lewidore, Lis, Sol. *Silver* : Crown, Gros, Lis, Petite Piece, Teſton. *Billon* : Cavalot, Denier, Douzain, Sol. *Copper* : Blank, Carolus, Denier, Double, Liard, Maille, Patac, Pite.

Germany. *Gold* : Ducat, Florin, Obolus, Rixgould. *Silver* : Florin, Hongre, Izelotte, Rixdollar. — *Billon* : Blaze, Ratze. *Copper* : Albus, Kreuxer, Pfenin, Plappert, Sexling, Rappen, Swaar, Tryling. V. *Netherlands*.

Italy. *Gold* : Piſtole. *Silver* : Carline, Croiſate, Derlingue, Ducatoon, Florin, Julio, Philip, Scudi, Teſtoon, Zecchin. *Billon* : Cavale, Papirole, Pignatelle. . . *Copper* : Bayoco, Quatrine.

Muscovy. *Gold* : Copec. *Silver* : Copec. *Copper* : Muſkofſke, Poluſke.

Netherlands. *Gold* : Albert, Crown, Ducat, Ducatoon, Florin, Imperial, Ride, Soverain. *Silver* : Florin, Gulden, Patagon, Philip, Schelling. *Billon* : Stuyver. *Copper* : Blanc, Duyt, Grooch , Penning, Stooter.

Poland. *Silver* : Abra, Groch, Ort, Roup.

Portugal. *Gold* : Joannes, Milrea , Moidore, Three-pound-twelvers. *Silver* : Cruzada, Pataca, Vintain. *Billon* : Vintain. *Copper* : Rez, Vintain.

Spain. *Gold* : Caſtellan, Doblon, Piſtole. *Silver* : Dollar, Piaſtre, Real. *Copper* : Blanca, Cornado, Ochavo, Quarto, Real.

Sweden. *Silver* : Caroline, Cavaliere , Chriſtin, Marc. *Copper* : Alleuvre, Dollar, Farthing, Marc, Mony, Rouſtique, Whitten.

DIFFERENCES

In the Britiſh and Foreign Coins.

ANGEL	*s*	*d*	*dec.*
1 Henry VI	6	8	
1 Henry VII	7	6	
34 Henry VIII & 6 Edw. VI	8		
ANGELET			
1 Henry VI	3	4	
CROWN			
Britiſh, 10 *James I*	5	6	
Engliſh . (V. *Thiſtle*)	5		
Florence	5	3	
France: *New*	4	10	·79
Old	4	6	
Genoa	6	6	·74
Milan	4	9	.08
Portugal	5	5	
Rome	5	3	·75
Thiſtle: 2 *Ja. I.*	4		
10 *Ja.* I.	4	4	·75
Venice	4	7	·50
of Coſmo III.	4	3	·73
DOLLAR			
Campen	2	7	
Dantzic	4	6	.29
Denmark: *Sletch*	6		
Embden	2	3	
Hamburgh	4	6	.78
Holland: *Leg*	4	4	.27
Lion	3	7	·72
Koningſburgh: *Bank*	1	—	—
Croſs	—	9	·72
Spain: *Croſs*	4	4	·75
Mexico	4	5	·79
Peru	4	5	
Pillar	4	5	·78
New Sevile	3	7	.80
Old Sevile	4	6	.15
Specie	2		
Sweden:			
Copper-plate	4	6	
Stuyver	2	2	.78
Thick	5		
Zeland	2	8	
DRITTLE			
Brandenb. & Lunenb. *New*	2	7	
Old	2	9	
DUCAT			
Gold. Arragon	6	6	
Barbary	9	4	·75
Hanover	9	2	.13
Hungary	6	4	
Poland	9	1	·55
Tranſylvania	8	11	.50
Silver. Naples	3	4	·44
Portugal	2	10	·33
Venice: *New, or Bank*	4	2	
Old, of Picoli, or Current	3	4	·50
DUCATOON			
Barcelona, Hamburgh	6		
Bergen	4	4	
Cadiz, Saragoſſa	5	6	
Cologn	5	5	.20
Flanders	5	6	.15
Holland	5	5	·59
Naples	5		
Valencia	5	3	
FLORIN			
Dry	5	4	

Geneva		6	.50
Cenoa		8	.25
Liege	2	3	
Norimberg	6	1	
Savoy		11	
Sicily	2	6	
Straſburg	1	8	
Venice	1	9	.60
GROAT			
Baſil			.88
Bremen			.50
Flemiſh			.56
French		1	.80
White		1	.75
GUILDER			
Baſil	4	8	.98
Brandenburg: $\frac{1}{3}$	1	1	.90
Gout (*Holland*)	2	2	.26
Hanover: $\frac{2}{3}$	1	2	.70
Hildeſheim	2	6	.21
Leopold	4	6	.28
Milan	2	3	.25
Norimberg	7	1	
Saxony: $\frac{2}{3}$	2	4	.12
Sigiſmund III.	4	6	.45
Thorn	4	5	.78
Uladiſlaus IV.	4	6	.48
Zell	2	3	.06
Zuric	4	4	.56
MARC			
Denmark: 4-*marc-piece*	2	8	.28
ſlet-marc	1	6	—
Italy	2	10	—
Lub, or Lubec	1	8	—
Saxon	6	—	—
Scotch	1	1	.50
Sweden: 4-*marc-piece*	2	6	.29
NOBLE			
England: *George*	9	9	.50
Ferling	1	8	—
Flanders	12	—	—

Maille	3	4	—
PATAGON			
Cologn	4	4	.53
Flanders	4	4	.91
Holland: *Leg*	4	4	.28
Liege	4	7	.48
PISTOLE.			
Italian, *Old*	16	7	.56
Spaniſh, *Old*	16	9	.30
RIAL			
Roſe	30	—	—
Speer	15	—	—
RIXDOLLAR			
Baſil	4	8	.24
Bavaria and Palatine	4	5	.78
Brandenburg, *Old*	4	7	.17
Cologn	4	4	.53
Dantzic, or Thorn	4	5	.85
Ferdinand, Duk of Auſtria	4	5	.78
Francfort	4	6	.53
Hamburg	4	6	.92
Hanover	4	7	.30
Leopold, and Ferdinand III.	4	6	.27
Liege	4	4	.20
Lubec	4	7	.54
Lunenburg	4	6	.65
Magdeburg	4	6	.27
Mentz	4	7	.27
Nuremberg	4	7	.55
Zurich	4	2	.65
SHILLING			
Lubeck	—	—	.37
Lucern	—	—	.57
Riga: *Black*	—	—	.05
Zercher, or Zurich	—	—	.75
SOVEREIGN			
England: 34 *H. VIII.*	20	—	—
2 *El.*	30	—	—
Great	33	—	—
Holland	2	3	—

TESTON

TESTOON.

Portugal	—	9	—
Rome	1	6	.84

UNITE.

10 James I.	22	—	—

DIVISIONS.

$\frac{1}{2}$ $\frac{1}{4}$ Angel. $\frac{1}{4}$ Angelet. $\frac{1}{2}$ Bayoco. $\frac{1}{2}$ (engliſh) Crown. $\frac{1}{2}$ $\frac{1}{3}$ $\frac{1}{4}$ $\frac{1}{12}$ $\frac{1}{16}$ (french) Crown. $\frac{1}{2}$ $\frac{1}{4}$ Dollar. $\frac{1}{2}$ Douzain. $\frac{1}{2}$ $\frac{1}{4}$ (gold) Ducatoon. $\frac{1}{2}$ $\frac{1}{4}$ Florence. $\frac{1}{2}$ $\frac{1}{4}$ Guinea. $\frac{1}{2}$ $\frac{1}{4}$ Lewidore. $\frac{1}{2}$ $\frac{1}{4}$ $\frac{1}{8}$ $\frac{1}{10}$ $\frac{1}{12}$ Moidore. $\frac{1}{2}$ $\frac{1}{4}$ Noble. $\frac{1}{2}$ Ochavo. $\frac{1}{2}$ $\frac{1}{4}$ Piſtole. $\frac{1}{2}$ Pite. $\frac{1}{2}$ Real. $\frac{1}{2}$ $\frac{1}{3}$ $\frac{1}{4}$ $\frac{1}{8}$ $\frac{1}{12}$ $\frac{1}{24}$ $\frac{1}{48}$ Shilling. $\frac{1}{2}$ Soverain. $\frac{1}{2}$ Sterling. $\frac{1}{2}$ Teſtoon. $\frac{1}{2}$ $\frac{1}{4}$ Vintain.

MULTIPLES.

2 (i. e. *Double*) britiſh Crown-piece (&c.) 2, 4, 6, 15, 30 Deniers. 2 Ducat. 1 $\frac{1}{4}$, 1 $\frac{1}{2}$, 2, 4, 8 (german) Groat. 2, 4 Lewidore. 2 Livre. 2, 2 $\frac{1}{2}$, 4 Moidore. 2 (elizabeth) Noble. 3, 4, 6, 8, 9 (german) Penning 2, 4 Piſtole. 2, 2 $\frac{1}{2}$, 4, 5, 10, 250, 400, 500 Rea. 2, 4, 6, 10, 20, 25 Sol. 2 Soverain. 10 Tarin. 2 $\frac{1}{2}$, 5, 12 Vintain.

SYNONYMS

Of Coins and Mony.

Alphonſi	*Maravedi*
Abras	*Brummer*
Aſlani	*Abouquel*
Bolognies	*Bayoco*
Byzantine	*Bezant*
Caſſa	*Rixdollar*
Caſtillian	*Caſtellano*
Cecchin	*Zecchin*
Cheffin, Chequin	*Zecchin*
Craca	*Grain*
Creux	*Kreuxer*
Croiſate	*genoa-Crown*
Cruiſade :	*ſpaniſh Crown, or Caſtillan*
Cruitſer	*Kreuxer*
Cruzada	*portugal Ducat*
Cuento (in Annotat.)	*V. Maravedi*
Dalle, Daller	*Dollar*
Demi-Angel	$\frac{1}{2}$ *Angel*
Demi bayoco	$\frac{1}{2}$ *Bayoco*
Demi-Maille	*Pite*
Dieci-Tarini	10 *Tarins*
Denain	*ſilver Copec*

Denier gros	*Penning*
Doublon	*Doblon*
ſtuyver-Dollar :	*Swediſh Rix-dollar*
turkiſh-Dollar	*Abouquel*
Douzain	*Sol*
gold-Ducat	*Hongre*
Duyt	*flemiſh Penny*
Eaſterling	*Sterling*
Ecu, Eſcu	*Crown*
Eſcalin	*Shilling*
Fenin	*Penny*
Franc	*Livre*
Genouin	*genoa Crown*
George-noble	*Noble*
Gilder	*Florin*
Gludſtadt	*Gluckſtadt*
Gould	*german Florin*
Grievener, Grieve	*Grif*
Gr-os, -och, -oſch	*Groat*
bohemia Gros	*Blaphace*
Guilder, Gulden	*Florin*
Harper	*iriſh Shilling*
Juſtine	*venice Ducatoon*
Juſtus judex	*Ebrew*
Kapeke	*Copec*
Laureat	*Carolus*
white Lewis	*french Crown*
Livre de gros	*dutch Pundt*
Louis-d'or	*Lewidor*
Lub	*Stuyver*
Lundreſs	*Sterling*
Malvedis	*Maravedis*
Manc-os, -uſa	*Mark*
Marabitini	*Maravedi*
Mearc	*Marc*
Mite	*Farthing*
Moeda d'oro	*Moidore*
Niquet	*Double*
new Noble	*Rial*
Obole	*Maille*
Obolus (in Annotat.)	V. *Penny*
Octavo	*Ochavo*

Par-a, -aſi, -at	*Meidein*
Pat-ar, -rd	*Stuyver*
Paullo	*Julio*
Penny	*Pfenin*
Peſo	*ſpaniſh Dollar*
Peſo d'oro	*Caſtellano*
Pezz-a, -o	*Dollar*
Pfenin	*Penny*
Philip	*Ride*
Philip of Milan	*Crown*
Piaſtre	*Dollar*
ſpaniſh Piſtole	*Doblon*
Piece of 8 (reals)	*Dollar*
Pogeria, Poitevin	*Pite*
Pougeoiſe	*Pite*
Poy	*flemiſh Penny*
Pundt	*Pound*
Quadrans (in Annot.)	V. *Penny*
Quadrine, Quartrin	*Quartile*
double Quarto	*Ochavo*
Quento (in Annot.)	V. *Maravedi*
Quilo	*Julio*
Ree, Rez	*Rea*
Real	*Rial*
Real of 8	*Dollar*
double Rial	*Sovereign*
Riſer	*half Purſe*
Royal	*Riall*
Runſtick	*Rouſtique*
Scalin, Schelling	*Shilling*
Sceptre	*Unite*
Scherif	*Zecchin*
Scudi	*Crown*
Seguin, Sequin	*Zecchin*
Sheckeen	*Zecchin*
Semipite	½ *Pite*
Seventeener	*german Florin*
Sixain	½ *Douzain*
Sol	*Bayoco*
Sol de gros	*dutch Schelling*
Sol-lub	*Schelling*
St. Stephen	*Millrea*

Sultanin	*Zecchin*	Toſtao	*Teſton*
Thaler	*Dollar*	Xerif	*Zecchin*

ANNOTATIONS
Upon the whole.

CASTELLANO—formerly, when it was ſtandard, and weigh'd 90 grains, was worth 598 $\frac{2}{3}$ maravedis——Since the year 1652, it has been raiſd to 739 maravedis.

CROWN (engliſh) goes—at Amſterdam, for 58 ſtuyvers, or ƒ 5 2 3 .04——at Rotterdam, for 58 $\frac{1}{2}$ ſtuyvers, or ƒ 5 3 1 .20. V. Shilling.

DENIERS are only current in the Southern part of France, there being none of them to be ſeen on this ſide Poitiers.

DOUBLES (that is, 2 deniers) ſometimes paſs for *liards*.

FLORINS *of Gold* are (moſt of them) of a very coarſe alloy; ſome of them not exceeding 13 or 14 carats; and none 17 $\frac{1}{2}$: they weigh about 2 dw. 13 gr.

In FRANCE, among traders, it is uſual (to ſave the trouble of telling their mony) to put it into bags. Theſe, in great dealings, they only weigh: and, if the receiver finds bad mony in the bag when he comes to open it, the payer makes it good: but, in caſe of a deficiency in the Sum, it muſt be challeng'd at the ſcale; otherwiſe it is not recoverable.

GOLD *Coins* (9 Jam. I) were raiſd by proclamation, 2 s. in every 20 s.

GROAT. In the Saxon times we had no ſilver mony bigger than a penny; nor, after the conqueſt, till Edward III: who, about the year 1351, coin'd *groſſes* (that is, groats, or great pieces) which went for 4 d a piece. And ſo the matter ſtood till the reign of Henry VII; who, in 1504, firſt coin'd Shillings.

GUINEAS were *firſt coin'd* by K. Charles II (ann. 1663), on the footing of 20 s: though they never went for ſo little. (V. Alloy). They have ſince advanc'd to 21 s. 6d: and, in the reign of K. William, were current at 30 s; people being willing to take them at any rate, rather than run the hazard of the ſilver-mony; which was then ſo exceedingly clipt, and counterfeited—— But, in 1696 (after the clipt mony ny was recoind) they came to ſettle at the price of 21s. 6d, and, at that rate, they continued to paſs, till they were brought down to their preſent value of 21s. by the proclamation of dec. 22. 1717.

In ITALY, the current mony (calld *imperial*, and *piccoli*) is 20 per cent worſe than bank-money (calld *di ova*).

LOUISD'ORS were firſt coin'd in 1640, under the reign of Louis XIII; and were valued at 10 livres; afterwards at 11, 12, 14. In the latter end of Louis XIV, they roſe to 20; and in the beginning of Louis XV, to 30, 36, and even to 40, and upwards; with this difference however, that, in the laſt coinings the weight was augmented in ſome proportion to the price; which, in the former reign, was never regarded.——The new louid'or is now valued at 20s 3d 3q.

MARAVEDIS have been of *different Values* in different ages. (1) In 1401, a maravedi was the $\frac{1}{3}$ of a real. (2) Under Alonço XI, it was worth 17 times as much as now. (3) Under Henry II, and John I, 10 times the preſent value. (4) Under Henry III, 5 times. (5) Under John II, 2 $\frac{1}{2}$ of the preſent. Mariana.——Anciently, *in Caſtile*, they reckoned by maravedis, and quentos. A *cuento* (or quento) of maravedis was 1.000,000 of them. Vayrac.

MARK *Scotiſh*, in K. Henry VIII's time, was valued at 3s 4d.

PENNY *(Denarius)* in our ancient hiſtories, is a term frequently uſd to denote an Integer: of which the *obolus* was half; the *quadrans*, a fourth.

PESO——of 10 reals, at Madrid, is 41 $\frac{7}{8}$ d——of 6 livres, at Leghorn, is 50 $\frac{1}{4}$ d. ——of 7 livres, at Genoa, is 53 $\frac{1}{2}$ d.

PIASTRES, ſtruck in *Mexico*, are ſomething heavier than thoſe ſtruck in *Potoſi*: But, in return, they are not quite ſo fine.

PURSE (without any addition) is to be underſtood of *ſilver*; thoſe of *gold* being ſeldom uſd, but for preſents to favorites.

SHILLING (engliſh) goes —in Holland, but for 10 $\frac{1}{2}$ (d 11 1 .36) or 11 ſtuyvers (d 11 3 .63). V. Crown.

In SICILY, *Malta*, &c. their money is made, the one of *ſilver*, the other of *copper* or braſs; the latter being their current mony: and, in buying or ſelling of any commodities of value, it is always ſaid whether of ſilver, or braſs-mony; the former being counted 50 per cent better than the latter.

Sol——*of gold*, in the time of the ſalique law, was 40 deniers——*of billon*: was firſt ſtruck on the foot of 12 deniers tournois; whence it was alſo calld douzain; though it went (afterwards) for 15; and, in 1709, was raiſd to 18.

In Spain they have new and old money: The *old* (current in Andaluſia, Cadiz, Seville, and ſome other places) is worth 25 per cent more than the *new*.

Testons were *firſt ſtruck* under Louis XII, in 1513; their value 10 ſols. Afterwards they roſe from 15 ſols, 6 den. to 19 ſols 6 den.

Tin *Farthings*, and *Halfpennies* were coind by K. James II, in 1685; and, in 1689, about 1.000,000*l*, in half crowns, ſhillings, and ſix-pences of old *Braſs guns, and utenſils of the moſt Refuſe metals*; when he landed, with french forces, at Kingſale in Ireland, on the 12th of march; the greateſt part of that kingdom (then) ſubmitting to him. And, before he left Ireland, a proclamation was preparing for the currency of *Pewter*-mony, and even of *Lead*; of which were coin'd ſome pence, and half-pence.

MEASURES.[a]

i. Applicatory.

Arſcheen, Brace, Cane, Cavado, Derſch, Ell, Pico, Ras, Shock, Toiſe, Vara, Veſchcove, Yard. V. *Proportions of meaſures*, p.

ii. Cubic.

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[a] *The buſineſs of meaſures, the great* Mr Chambers *obſerves, has been ſo confuſedly, and (withal) ſo imperfectly deliverd by our Engliſh writers, that his readers, he preſumd, would not be diſpleaſd with the pains he had taken to diſembroil what was perplext, and to ſupply what was wanting—How-many* particulars *I have added to his account, and how-much more light I have derivd on the ſubject by the* diſpoſition *of them; I leave to the obſervation of the curious; and earnly pray them (as, in the view, in which I have here preſented the doctrin, it will be much eaſier than ever to ſee what is erroneous, or defective) to aſſiſt me with ſuch hints as may occur to them, for the improvement of this article: which (after all) I am ſenſible is ſhort of the perfection it is capable of.——* *NB.* The proportion *of the foreign meaſures* to thoſe of Britain, *when not ſpecified in the Notes, may be found under the head of proportions of meaſures, p.*

ii. CUBIC.

1. *Dry*[b].

Almud 11. Alquie 8. Aſnée 3. Bochel 6. Boiſſeau 3. Charge 3. Chetweric 7. Chetwert 7. Corbe 6. Emine 3. Fanega 11. Halſter 5. Hoedt 5. Killow 12. Laſt 4 5 9. Litron 3. Load 3, Meſurette 3. Mina 6. Mine 3. Minot 3. Moggio 6. Mouver 5. Moy 6. Mude 5. Muid 3. Muken 5. Picotin 3. Pipe 3. Quartier 3. Quartillo 11. Raiſer 5. Rugio 6. Sac 3 5 6. Salma 11. Salmo 6. Scheppel 4 5 9. Setier 3. Staro 6. Stax 6. Tun 3 5 10. Viertel 2. Voye 3.

1. DENMARK, V. Scandinavia.——MOREA, V. Italy. NORWAY, V. Scandinavia.——SICILY, V. Italy.——SWEDEN, V. Scandinavia.

2. FLANDERS: Viertel. V. Proportions.

3. FRANCE: [*Corn, &c.*] Quarte 2 Litron[c] 4 Picotin 4 Boiſſeau[d] 3 Minot[e] 2 Mine[f] 2 Setier[g] 12 Muid[h].——DIFFERENCES (1) *Char-coal:* Litron 2 Picotin 4 Boiſſeau 8 Minot 2 Mine 16[i] Muid. (2) *Lime:* Boiſſeau 3 Sac 36 Muid[k]. (3) *Oats:* Buſhel = double that of any other grain. (4) *Plaſter:* Boiſſeau 12 Sac 36 Muid.

(5)

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[b] *The following alphabetical* indexes *of meaſures and weights are here given (for the readier recourſe) with references, by figures, to the account given of them in the ſubſequent paragraphs; which are, therefore, diſtinguiſht by numbers prefixt.*

[c] *By ordonnance, the* litron *is divided into 2 demi-litrons, and 4 quarter-litrons; and contains 36 cubic inches of Paris.*

[d] *By ordonnance, the Paris* buſhel *is to be 8 inches, 2 lines, and a half high; and 10 inches broad (or, in diameter) within-ſide.—But it is different, in different juriſdictions. V. Oats.* (3)

[e] *By ordonnance, The* minot *is to be 11 inches 9 lines high; and 14 inches 8 lines in diameter.—It commonly weighs* 240 lb, *marc-weight.*

[f] *The* mine *is no real veſſel; but an eſtimation of ſeveral others.*

[g] *A* ſetier *of good wheat weighs betwixt* 244 *and* 248 lb, *marc-weight.——In Berri the ſetier conſiſts of* 16 *boiſſeaux.*

[h] *The* muid *of Paris is to weigh* 2640 lb: *that of Orleans* 600 lb.—*That of Berri is* 21 *boiſſeaux.*—*That of Rouen is* 12 *ſetiers, which makes* 14 *of Paris; and weighs* 3360 lb.——*And is divided into mines: but thoſe mines only contain* 2 ½ *paris-ſetiers.*

[i] *Inſtead of* 16 *mines to the* muid, *the city-meaſure gives* 20.

[k] Or 3 *boiſſeaux is* 1 *Minot; and* 48 *minots,* 1 *Muid.*

(5) *Salt* [l]: Meſurette 16 Litron [m] 16 Quartier 4 Minot [n] 4 Setier 10 Muid. (6) *Sea-coal:* Litron 4 Picotin 4 Boiſſeau 6 Minot 15 Voye. (7) *Wood:* Boiſſeau 8 Minot 2 Mine 20 Muid——VARIATIONS (1) *Aſnée* (of Maçon = $1\frac{1}{3}$ ſetier. (2) *Emine* (of Caſtres) = $\frac{1}{2}$ a ſetier [o]. (3) *Load* of Tarſchoon = lb 352. 80: of Arles, 360: of Beaucaire, 367. 20: of St. Gilles, 428. (4) *Pipe* (in Bretagne) = 10 charges, each of 4 boiſſeaux; and weighs 600 lb. (5) *Tun* [p] of Auray = lb 2200: of Breſt, Port-louis, Quinpercorentin, 2240: of Audierne, 2300.

4. GERMANY: Scheppel [q] 90 Laſt.

5. HOLLAND: Scheppel 4 Mude 27 Laſt [r]——DIFFERENCES: *Oats:* Laſt (at Berg-op-zoom) 28. 25 viertels. (at Bruges) 14. 50 hoedts. (at Dixmuyde) 24 raiſers. (at Ghent) 19 ſacs, or 38 halſters. (at Gravelin) 18 raiſers. (at Liſle) 30 raiſers. (at Steenberge) 29 viertels. (at Tongres) 14 muids.——VARIATIONS: (1) *Halſter* = [muid] $\frac{1}{8}$ in Louvain. [ſac] $\frac{1}{2}$ at Ghent. (2) *Hoedt* = [mouvers] 8, in Bois-le-duc, Cleves, and Guelderland. [muids] 9 in Overyſſel, Zwell. [raiſers] 18, at Dunkirk. [ſacs] 10. 50, at Utrecht. 10.66, at Delf, Rotterdam. [viertels] 13 at Amſterdam. (3) *Laſt* = [hoedts] 17.50, at Bruges. [mouvers] 20.50, at Bois-le-duc. 22, in Guelderland. [muids] 8, in Liege [s]. 15, at Tongres. 25, at Utrecht. 33, in Frieſland. 36, at Deventer. 37, at Louvain. [raiſers] 22.58, at Gravelin,

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[l] Salt *is ſold, all over France, by the muid; which is greater, or leſſer, according to the cuſtom of the provinces, where it is made, and where it is ſold.—In Marenne in the iſle of Rhé, and other places in France, where ſult is made, they ſell it by the* hundred; *which they divide into 28 muids, and every muid into 24 buſhels: and the hundred commonly makes, in Amſterdam, 11 $\frac{1}{2}$ laſts, or 23 tuns.*

[m] *The* litron *for ſalt is larger than that for corn; and is divided into 2 halves, 4 quarters, and 16 meſurettes.*

[n] *The* minot *of ſalt commonly weighs* 240 lb, *marc-weight.*

[o] *The* ſetier *of Caſtres weighs about* 200 lb *weight of that place; which is about* 170 lb *marc-weight.*

[p] *The* tun *of Nantes conſiſts of* 10 *ſetiers; the ſetier, of* 10 *buſhels.. The meaſure.. being heapt, it weighs between* 2200 *and* 2250 lb.. *when ſtricken, it weighs* 18 *or* 20, *per cent, leſs.*

[q] *Of the* ſcheppels *of Hamburg,* 83 *make about* 10 *quarters in London.*

[r] *The* laſt *commonly weighs* 4000lb *weight—At Amſterdam, and in North holland, a Laſt.. of Barley, commonly weighs between* 3200 *and* 3400 lb . . *of Rye:* 400 *and* 4200 . . *of Wheat:* 42000 *and* 4800.

[s] *Where they reckon* 12 *ſetier to a* muid, *and* 8 *muids to a laſt.*

St. Omers. 30·50, at Dixmuyde. 41, at Liſle. [ſacs] 4·50, at Middleburg [t], in Zeeland. 25, at Bruſſels, Utrecht. 26, at Alcmaer. 28, at Tergou. 29, at Delf, Rotterdam, Schedam [u], and Ghent. 33, at Haerlem. 36, at Amſterdam, Montfort, Vianen, Yſelſtein [w]. 44 [x], at Enchuyſen, Hoorn, Muyden, Naarden, Weſop. [ſcheppels] 33, at Thiel. 68, at Ruremond. 90, at Hamburg. [viertels] 28, at Mechlin. 33·50, at Breda, Steenberge. 34, at Berg-op-zoom. 37·50, at Amſterdam. 38, at Antwerp. (4) *Mouver* = 4 Scheppel. (5) *Muken* = $\frac{1}{4}$ of a viertel. (6) *Raiſer* = [ſcheppels] 2, at St. Omers. (7) *Sac* [muid] $\frac{1}{6}$, at Ghent. [ſcheppels] 2, at Enchyſen. 2.25, in Zeeland. 2.50, at Brill, Fluſhing, Zurickſee. 3; at Amſterdam. 8, at Leyden. [viertels] 32, at Antwerp.

6. ITALY: *Bologna:* Corbe [y]. *Florence:* Staro [z] 24 Moggio. *Genoa:* Mina 20 Tun [a]. *Leghorn:* Stax 3 Sac [b] 8 Moggio. *Morea:* Bochel [c]. *Rome:* Rugio [d]. *Sicily:* Salmo [e]. *Venice:* Staro [f].

7. MUSCOVY: Chetweric 8 Chetwert [g].

8. PORTUGAL: Alquie 60 Moy [h].

9. PRUSSIA: Scheppel 60 Laſt [i].

10. SCANDINAVIA: Tun [k].

11. SPAIN: *Biſcany:* Fanega [l]. *Catalonia:* Quartille [m] 4

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[t] *Where the* ſac *is little more than* 2 *ſcheppels.*

[u] *The* laſt *of theſe places is* 2, *per cent, more than the laſt of Amſterdam.*

[w] *Where they reckon* 2 *ſacs to a muid.*

[x] *Each* ſac *containing* 2 *ſcheppels.*

[y] *Of the* corbes, 100 *make about* 92 *ſetiers of Paris.*

[z] *The* ſtaro *weighs about* 50 lb. —*Salt is ſeld by the ſtaro of* 72 lb.

[a] *The* tun *of Genoa is about* 40 *buſhels of wincheſter meaſure.*

[b] *At Leghorn* 5 *ſacs is accounted to make little leſs than* 10 *engliſh buſhels.*

[c] *Of the* bochels, $9\frac{2}{3}$ *make about* 8 *buſhels wincheſter-meaſure.*

[d] *The* rugio *weighs about* 412lb; *and makes, in Florence,* $2\frac{1}{2}$ *ſtaros.*

[e] *Of the* ſalmo, *they have* 2 *ſorts; viz. a great ene, and a ſmall one; which is the common one: and is about* $7\frac{1}{9}$ *buſhels, wincheſter-meaſure; and about* 17, *per cent, leſs than the great one.*

[f] *The* ſtaro *of Venice is about* 1 *ſetier of Paris.*

[g] *The* chetwert *holds about* $5\frac{3}{20}$ *buſhels of woncheſter-meaſure.*

[h] *A* moy *contains about* 3 *engliſh quarters.*

[i] *Of the Pruſſia* ſcheppels, 4 *make a muid; which is a ſtone of* 34 lb.

[k] *The* tun *of Denmark and Norway is about* 4 *London-buſhels.*

[l] *Of the* fanegas *of Biſcany and Valencia,* 5 *make an engliſh quarter.*

[m] *The* quarter *of Catalonia is the ſame as the engliſh quarter: and* $2\frac{1}{2}$ *of them is a cargo, or load, of* 360 lb *weight.*

4 Salma. *Granada:* Almude [n] 2 Fanega. *Valencia:* Fanega [o].

12. TURKEY: Killow [p].

2. *Liquid.*

Aem 1 3 4. Achtelin 3. Açumbre 9. Amphora 5. Anker 4. Arroba 9. Aſnée 2. Barile 5. Barique 2. Baſſee 5. Beſon 3. Bigorza 5. Boccale 5. Botte 2. Brenta 5. Buſſard 2. Cantar 9. Cavada 6. Chopine 2. Cogno 5. Demi-ſetier 2. Driolinck 3. Fat 7. Feoder 3 4. Feuillette 2. Fiaſco 5. Heemer 3. Je 3. Kan 8. Lera 5. Levor 5. Loder 5. Maſſem 3. Maſtilli 5. Mero 5. Meter 20. Millerole 2. Miſerole 5. Mittidel 5. Mizàro 5. Muid 2. Muſſie 4. Oke 10. Oxio 5. Pignatoli 5. Pinte 2 4. Pipe 1 2 9. Poinçon 2. Poiſſon 2. Pot 2 6 8. Quarta 5. Quartillo 9. Queuë 2. Reoder 3. Rubbia 5. Sachie 5. Salma 5. Scandale 2. Schreve 3. Sechys 5. Seiltin 3. Setier 2. Somar 9. Staio 5. Steckan 4. Stoop 1 4 7. Tomolo 5. Tun 2 4 5. Viertel 3 4. Urna 5. Yune 3.

1. FLANDERS: Stoop [q] 50 Aem [r] 3.04 Pipe.——DIFFERENCES: *Beer* is ſold by the barrel of 52 ſtoops. The Stoop contains 7 pints: and the Aem is about 42 gallons wine-meaſure, in London.

2. FRANCE: Poiſſon [s] 2 Demiſetier 2 Chopine [t] 2 Pinte [u] 2 Pot 4 Setier [w] 36 Muid [x] 3 Tonneau.——DIFFERENCES: *Brandy*

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[n] *An* almude *is about* $1\frac{1}{2}$ *engliſh buſhels: in weight . . by heap,* 144 lb . . *by ſtrike,* 99 lb *engliſh.*—NB. 100 ſacs *of Granada make . .* 3 *laſts* 10 *muids of Amſterdam . .* 64 *ſetiers of Paris . .* 128 *buſhels of Bourdeaux.*

[o] *The* fanega *of Valencia makes about* 2 *engliſh buſhels.*

[p] *The* killow *weighs about* 20 *okes: and* $8\frac{2}{3}$ *of them make about* 1 *engliſh qnarter.*

[q] *The* ſtoop, *they reckon, weighs* 6 lb *of their weight.*

[r] *Six* aems *make a tun of* 252 *gallons, wincheſter-meaſure.*

[s] *The* poiſſon *contains* 6 *cubic inches.*

[t] *The* chopine *of common water weighs* 1 lb *of Paris.*

[u] *The* pinte . . *of Paris is about the 6th part of the roman congius; and contains* 2 lb *of common wator. That . . of St Denis is bigger.—In ſeveral places it is called a* pot.

[w] *The* ſetier, *among gaugers, is an eſtimation of* 8 *pintes.*

[x] *The* muid—*at Bourdeaux, contains* 110 *pots with the lee, or* 100 *pots clear meaſure; the pot containing about* 2 *mingles of Amſterdam.—of Champaign* = 48 *ſetiers.—of Montpelier and ſeveral other places in Languedoc,* = 18 *ſetiers; the ſetier* 32 *pots, equal to* 35 *ſteckans of Amſterdam.—of Paris, according to an ordonnance of Henry IV,* 300 *pintes; according to the regulation of Louis XIII,* 280 *pintes.*

Brandy (at Montpelier, Tholouſe, &c.) is ſold by the quintal of 100 *lb* weight with the caſk——VARIATIONS: (1) *Aſnée* (at Lion) 80 pots. (2) *Barique* (at Bourdeaux) ¼th of a tonneau, or 3 paris-muids. (3) *Botte* (in the Southern provinces) a veſſel containing about a muid. (4) *Buſſard* (in Anjou and Poitou) half a pipe. (5) *Feuillette* (in Burgundy) 120 pintes (in Some provinces 1 ½ a pinte.) (6) *Millerole* (in Provence) about 66 pintes of Paris. (7) *Pipe*[y] (in Anjou and Poitou) a muid and half of Paris, or 432 pintes. (8) *Poinçon* (at Paris) a demi-queue, or half a queue. (in Blaiſois and Touraine) half the orleans-tun, or 288 pintes. (at Rouën) 13 boiſſeaux. (9) *Queuë* (in Champagne, and at Orleans) 54 ſetiers. (10) *Scandale* (in Provence) 4 ½ gallons wincheſter-meaſure. (11) *Tonneau* (at Bayonne and Bourdeaux) 3 paris muids, or 864 chopines. (at Berri, and Orleans) near 2 paris muids, or 576 chopines.

3. GERMANY: Maſſem 4 Viertel 20 Aem 6 Feoder[z] 2.50 Reoder——VARIATIONS.. *Augſburg:* Maſſem 8 Beſon 12 Je[a] 8 Feoder.. *Heidelberg:* Maſſem 4 Viertel 12 Aem 10 Feoder.. *Nuremberg:* Maſſem 64 Heemer 12 Feoder.. *Vienna:* Seiltin 4 Achtelin 32 Heemer 32 Feoder.. *Wirtemberg:* Maſſem 10 Yune 16 Aem 6 Feoder.—*NB.* (1) *Driolinck* (of Vienna) = 24 heemers. (2) *Schreve* (of Vienna) = 4 maſſems.

4. HOLLAND: Muſſie 4 Pint 2 Mingle 2 Stoop 8 Steckan 2 Anker 4 Aem 14 Feoder.——DIFFERENCES: (1) *Hony* (at Amſterdam) is ſold at ſo-many pounds flemiſh per tun; conſiſting of ſix aems.. or by ſo-many florins per barrel.. or by the hundred-weight. (2) *Oils* are ſold by the tun; which contains 6 aems, equal to 1600 paris pintes... Coarſe fiſh-oil they commonly keep in barrels, containing from 15 to 20 ſteckans.——VARIATIONS: (1) *Viertel*, or *Virge*[b]: 5 ⅙ mingles.. that of Wine, 6 mingles.. that for Brandy, 6 ¼. (2) Tun, V. Differences: Hony, Oil.

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[y] *The* pipe *goes by the name of* tun *in the provinces beyond the Loire.*

[z] *The* feoder *is ſuppoſd the load of a wagon with 2 horſes.*

[a] *The* je *is* 12 *beſons, or* 3 *muids.*

[b] *At Bruges they call the* virges *Seſtiers; reckoning* 16 *ſtoops to a ſeſtier.*—NB. *It is common to put French, Spaniſh, and Portugal wines into pipes, butts, and other pieces; ſome of which will contain, at Amſterdam,* 60 *to* 90 *virges. It is uſual, therefore, with the Hollanders, to reduce thoſe meaſures into butts by the following reckoning:* For one Butt, *Virges* .. 27, *at Cognac, Embden, Monguion, Rhé, Rochelle* .. 29, *at Nantes, and other places in Bretagne and Anjou* .. 30, *at Hamburgh, and Lubeck* .. 32, *at Amſterdam, and other places in Holland; and at Bourdeaux, and other places in Guienne.*

5. ITALY: *Calabria:* Pignatoli [c] 32 Staio 10 Salma. *Ferrara:* Sechys 8 Maſtilli. *Florence, & Leghorn:* Mittidel 2 Fiaſco 20 Barile 10 Cogno. *Genoa:* Barile 2 Miſerole 5 Botte [d]. *Iſtria:* Sechys 6 Urna. *Morea:* Lodere. *Rome:* Boccale [e] 7.50 Rubbia 13.50 Brenta. *Venice:* Lera 4 Sachie 4 Quart 4 Bigorza 4 Amphora [ee]. *Verona:* Baſſee 16 Brenta.——DIFFERENCES: (1) *Corn* (at Venice) is ſold by the ſtaio; and is, in quantity, about 1 ſetier of Paris. (2) *Oil* (at Genoa) Barile 14 Tun [f]. (at Leghorn) Mittidel 32 Oxio. (in Morea) Levor [g] 10 Barile. (at Naples) Tomolo 16 Salma [h]. (at Venice) Mero [i] 40 Mizaro.

6. PORTUGAL: Cavada 12 Almuda [k].——VARIATIONS: *Pots* are alſo uſd, holding . . ſome ½ a gallon . . others ¼ of a gallon.

7. PRUSSIA: Stoop 180 Fat [l].

8. SCANDINAVIA: Pot 4 Kan [m].

9. SPAIN: Quartillo 4 Açumbre [n] 8 Arroba [o]. *Andaluſia:* Quartillo 4 Somar 8 Arroba. *Valencia:* Cantar [p].——DIFFERENCES: *Oil* [q]*:* Pipe 40.50 Arroba.

10. TURKEY: Oke 8 Meter [r].

WEIGHTS [s].

Ace 4. Arrata 7. Arroba 5, 7, 10. Baleman 11. Barquit 6. Cantar 5, 11. Carat 2, 4. Cargo 10. Caſtillan 10. Centner 3, 8, 9. Denier 5. Dram 2, 4, 5, 7, 10, 11. Engel 4. Grain 1, 2, 4, 5, 10, 11. Gros 4, 5. Killet 11. Laſt 8, 9. Liſpound 3, 4, 8, 9. Livre 2. Load 4. Loodt 3; 4. Marc 2, 4, 5, 10. Medical 11. Millier 2. Numile 3. Obolo 5. Ochavo 7. Octave 5. Oke 5, 11. Ounce 1, 2, 3, 5, 7, 10, 11. Penny-weight 1. Pood 6. Pound 1, 3, 4, 5, 7, 8. Primi 5. Quarta 5. Quint 3. Quintal 1, 2, 5, 7, 10. Quirat 11. Rota 11. Rotello 5. Rugio 5. Saliqua 5. Schippound 3, 4, 8, 9. Seſtertia 3. Scrupule 2, 5. Shock 8. Stone 3, 4, 8. Tomine 10. Tun 3. Vicomté 2. Wage 4. Zollotnic 6.

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[c] *The* pignatoli *is equal to the french* pinte.

[d] *The* botte *is about* 100 *pints*—*The Venetian botte is divided into muſtaches*; 76 *whereof make the amphora.*

[e] *The* boccale *contains ſomewhat more than a paris pinte.*

[ee] *By wholeſale, the* amphora *is* 14 *quarts*; *and the* bigorza, 3 ½ *quarts.*

[f] *The* tun *of Genoa makes* 236 *London-gallons.*

[g] *The* levor *weighs* 7 ½ lb: 10 *whereof fill a candia-barile*; *which ſhould hold* 15 *engliſh gallons, and weigh* 112 ½ lb *averdupois.*

[h] 5 ½ ſalmas *are accounted to a tun of* 236 *gallons of oil in London.*

[i] *The* mero *is about* 3 ⅓ *pints by meaſure*: *but, by weight, it is more.*

[k] *The* almuda *holds* 4 ½ *gallons wincheſter meaſure.*

[l] *The* fat *is about* 80 *ſtoops antwerp-meaſure.*

[m] *The* kan *holds near* 1 *gallon wincheſter-meaſure.*

[n] *An* açumbre *is about* 3 *pints engliſh.*

[o] 81 arrobas *in Andaluſia, make* 252 *gallons engliſh . . but, upon the Seville-gauge,* 236.

[p] *The* cantar *holds about* 2 *gallons wincheſter-meaſure.*

[q] *The meaſure for* oil *is the arroba of* 25 lb. *Hayes, negotiator*'s *magazine, p.* 350.

[r] *The* meter *is about* 2 *thirds of a wincheſter-gallon.*

[s] *In moſt parts of Europe the weights for coarſer commodities are* different *from thoſe, with which they weigh gold, ſilver, jewels, &c.*

1. FLANDERS: Ounce 16 Pound 100 Quintal.—TROY: Grain 32 Penny-weight 20 Ounce 8 Pound[t].

2. FRANCE: Livre 100 Quintal[u] 10 Millier.—TROY[w]: Karat 24 Grain 24 Scrupule 3 Drachme 8 Once[x] 8 Marc 2 Livre[y].—— DIFFERENCES . . . *At Calais* they have three ſorts of weights: (1) The firſt is the *town-weight*; 100 lb of

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These latter I ſhall take-notice of under the title of troy-weight.—In the mean time it muſt be obſervd that there are in ſome places, ſtill further diſtinctions of weights: Thus . . at Genoa *they have 5 kinds of weights: (1) Large weights, by which all merchandizes are weighd at the cuſtom-houſe (2) Caſh-weights, for piaſtres, and other ſpecies. (3) The Cantara or quintal, for the coarſeſt commodities. (4) The Large balance, for raw ſilks; and (5) The Small balance, for finer commodities.*

[t] 112 *flemiſh*-pounds *make about* (100 lb) *troy-weight of London.*

[u] *In ſome places a difference is made between a* quintal *and a* hundred-pound weight: *ſo that the buyer and ſeller are obligd to explain themſelves on this head in making of bargains.*

[w] *Beſide* (1) *The* poids de Marc, *generally uſd by merchants for things in little compaſs*; (2) *In Provence and Languedoc they have the* poids de Table; *which is found to differ, in ſeveral places, from the marc-weight,* 18, 20, *or* 25 *per cent:* 16 *ounces of the table being ſcarce* 14 *of the marc.* (3) *At Rouën they have another ſort of weight, calld the* poids de Vicomté; 100 lb *of which is reckond to make about* 108, *or* 110 lb *marc, or paris-weight*; *eſpecially in weighing of wool: but, in weighing other things, the ſaid* 100 lb *is counted* 104 lb *marc-weight* *. (4) *In Languedoc, they have alſo another ſort of weight, calld the* Roman *or* Statera-weight; *which is much the ſame as the table-weight.*

* *Of the Vicount-weight, they have no leſs a weight than* 13 lb: *ſo that all goods, weighing under that weight, are weighd by the marc-weight.*

[x] *The* ounce—*with Goldſmiths and Monyers, is divided as follows, viz. Grain* 7.20 *Felin* 2 *Maille* 2 *Eſtelin* 20 *Once.*————*with Phyſicians, is the* 12*th part of the livre. V. Livre* y.

[y] *The* livre—*of Avignon, Lions, Montpelier, and Toulouſe, is* 13 *ounces: that—of Rouën,* 14: *that—of Marſeille, and Rochelle,* 19. *V. Ounce* x.

of which makes about 92 lb in London. (2) The ſecond is the *merchant*'s *weight*; of which 100 lb makes 113 in London. (3) The third is calld the *engliſh wool-weight*; and is about 3 per cent lighter than the town-weight . . . *At Lions*, the weights are of two ſorts: (1) The *city-weight*, which weighs 14 ounces of the pound de marc, for common uſe; and (2) The *other* of 15 ounces, for ſilk. NB. 100 lb ſilk-weight makes 108 of the city-weight: and 100 lb of the city-weight is about 94 lb avoirdupois engliſh . . . *At Rouen*, they have two ſorts of weight: viz. *de vicomté*, and *de marc*. 108 lb of the former makes 100 of the latter, or 110 lb of paris-weight[z].

3. GERMANY: Loodt 2 Ounce 16 Pound[a] 14 Liſpound[b].——TROY: Seſtertia 4 Numile 4 Quint 4 Loodt 16 Marc[c].——VARIATIONS: (1) *Centner*[d] = 8 liſpound. (2) *Schippound* of feathers and wool is 20 liſpound. (3) *Stone* . . of flax, 20 lb . . of wool and feathers, 10 lb. (4) *Tun* of butter and tallow is 16 liſpound.

4. HOLLAND: Grain 4 Dram 3 Gros 2.50 Engel 10 Loodt 16 Marc 2 Pound 8 Stone.——TROY: (1) *For weighing Groſs gold:* Ace 32 Engel 20 Ounce 8 Marc. (2) *For Fining of gold:* Parts 24 Grain 12 Carats 2 Marc[e].——DIFFERENCES: *Salt* (at Amſterdam) is ſold by a great hundred of 404 ſcheppels; which is reckoned 7 laſts, or 14 tuns, or 28000 lb: which is alſo counted 208 ſacs; and is ſold by the pound flemiſh: and 11 $\frac{1}{2}$ laſts of Amſterdam make about the great hundred in the iſle of Rhé in France.——VARIATIONS: (1) *Liſpound* = 15 pounds. (2) *Load* = 400 lb. (3) *Schippound* = 20 liſpound. (4) *Wage* = 165 lb.

5. ITALY: Ounce 12 Pound 25 Arroba 6 Quintal. *Morea:* Dram 11.11 Ounce[f] 12 Pound 3 Oke 44 Quin-

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[z] *This laſt is only to be underſtood in the weighing of wool: or elſe* 100 lb *marc-weight weighs but* 104 lb *of the viſcount-weight. V. nete* w.—*And* 100 lb *de vicomté is about* 113 $\frac{1}{3}$ lb *avoirdupois engliſh.*

[a] *The* pound *of Vienna* . . *in ſome commodities, is divided into* 32 *loodts* . . *in others, into* 28 *pints.*

[b] *A* liſpound *of feathers, or wool, is* 16 lb.

[c] 116 marcs *make* 100 lb *troy-weight in London.*

[d] *A* centner *makes about* 120 lb *avoirdupois in London.—The centner of tin, at Dantzic (which conſiſts of* 120 lb*) makes, in London, about* 112 lb.

[e] *The* marc-weights *of Holland are about* 1 *per cent lighter than the troy-weight of London.*

[f] *In weighing* raw-ſilk, *they reckon* 15 *ounces to the pound.*

tal. *Sicily:* Ounce 12 Pound 2.50 Rotello 100 Cantar [g]. *Venice:* Pound 100 Quintal [h].——TROY: *Genoa, Florence, & Leghorn:* Grain 24 Denier 3 Groſs 8 Ounce 8 Marc [i]. *Naples:* Octave 8 Ounce 12 Pound [k]. *Rome:* Primi 4 Saliqua 3 Obolo 2 Scruple 3 Dram 8 Ounce 8 Marc [l]. *Venice:* Grain 4 Saliqua 9 Quarta 4 Ounce 8 Marc [l].—DIFFERENCES: *Corn,* or Grain (at Florence) is ſold by the Moggio of 24 ſtaios, of 50 lb weight, each.———VARIATIONS: (1) *Quintal* (at Leghorn) is of 4 ſorts: that of .. 100 lb, for Alum, 150 lb, for Sugar .. 151 lb, for Fiſh .. 160 lb, for Wool. (at Rome) of 2 ſosts: that of .. 160 lb, for Spices, and other choice goods .. 250 lb, for heavy Bulky goods; about 200 lb, avoirdupois in London. (2) *Rotello* (at Genoa) is 18 ounces. (3) *Rugio* (at Rome) is 412 lb of their weight; and makes (in Florence) 3 ½ ſtaios.

6. MUSCOVY: Zollotnic 96 Pound [m] 40 Pood 10 Barquit.

7. PORTUGAL: Dram 2 Ochavo 8 Ounce [n] 16 Arrata 32 Arroba 4 Quintal [o].

8. PRUSSIA: (1) *Groſs or bulky goods:* Pound 34 Stone [p] 10 Schippound. (2) *Fine goods:* Pound 16 Liſpound 20 Schippound.——VARIATIONS: (1) *Centner* of Tin (at Dantzic, 120 lb; at Koningſberg, 128 lb) makes, in Lon-

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[g] *The* cantar *is about* 176 lb *avoirdupois in London.*

[h] *The* Venice weights *are diſtinguiſht into groſs and ſuttle.* (1) *By the* groſs *they weigh coarſer metals, feathers, and other lumbering commodities.* (2) *By the* ſuttle *they weigh ſilk, ſpices, drugs, &c.* —*And* (1) 100 lb *groſs-weight, make* 158 lb *ſuttle*; *or* 106 lb *avoirdupois in London.* (2) 100 lb *ſuttle, make* 63.66 *groſs*; *or about* 65.75 *avoirdupois in London.*

[i] *Of the Italian* marcs, 100 lb *troy-weight, in London, make about* .. 130, *at Genoa* .. 158.50, *at Florence*—NB. 100 lb *troy-weight, is equal to* 86.50 lb *of their weight for ſilver*; *wherein they reckon* 12 *ounces to the pound.*

[k] 86.50 *Naples*-pounds *make about* 100 lb *troy-weight, in London.*

[l] 116.50 *Roman, Venetian, and Verona*-marcs *make* 100 lb *troy-weight, in London.*

[m] *The Ruſſia pound is counted equal to* 13 *oz* 3 *dw* 6 *gr. troy-weight*; *or* 14.15 *oz avoirdupois.*

[n] *The Portugal* ounce *is alſo divided into penny-weights and grains*; *the ſame parts the ounce troy is divided into, in London.*

[o] *The Portugal* weights *are between* 3 *and* 4, *per cent. heavier than the London avoirdupois.*

[p] *The little Pruſſia, and Dantzic*-ſtone *(for pepper, ſpices, and other fine goods)* *is* 24 lb.

[q] *The* laſt .. *of Dantzic, makes, in London, about* 17 cw 7 lb .. *of Koningſberg, a ſmall matter above* 18 cw.

don, 112 lb. (2) *Laſt* . . of hemp, flax, and ſuch light goods is 6 ſchippound, or 60 ſtone, viz. at Dantzic, 2040 lb; at Koningſberg, 2400 lb. [q]. But . . for Pot-aſhes, the laſtage is reckond double; viz. 12 ſchippound of them take-up no more room, in a ſhip, than 6 ſchippound of hemp or flax. (3) *Shock* is 60 pieces of any commodity ſold by tale [r].

9. SCANDINAVIA: Pound 20 Liſpound 20 Schippound [s]. ——VARIATIONS: (1) *Centner* of gun-powder is 120 lb. (2) *Laſt* of flax, hemp, cordage, and tallow, is 6 ſchippound.

10. SPAIN: Dram 8 Ounce 8 Marc 2 Pound 25 Arroba 4 Quintal. *Caſtile:* Grain 28 Dram 16 Ounce 16 Pound 25 Arroba 4 Quintal. *Catalonia:* Quintal [t] 3 Cargo. *Valencia:* Pound 24 Arroba 4 Quintal 2.50 Cargo. ——TROY: *Gold:* Grain 12 Tomine 8 Caſtillan 6.25 Ounce 8 Marc. *Silver:* Grain 75 Dram 8 Ounce 8 Marc [u]. ——DIFFERENCES: *Wool* is ſold by the arroba of 30 lb. And 100 lb of London makes about 102 lb in Catalonia: but it makes about 92 lb of their wool-weight.

11. TURKEY: Grain 4 Quirat 16 Dram 12 Ounce 12 Rota 150 Cantar [w].——TROY: Killet 24 Medical [x]. ——DIFFERENCES: *Silk* is ſold by the baleman, containing 6 okes [y].

SYNONYMS.

Aſchnee, V. *aſnée*. Aum, *aem*. Barrel, *brenta*. Bottle, *boccale*, *mingle*. Brante, *brenta*. Buſhel, *boiſſeau*, *ſtaio*. Butt, *pipe*. Carat, *ſaliqua*. Denier, *ſcrupule*. Dram, *medical*. Fat, *pipe*. Feillette, *feuillette*. Fertel, *viertel*. Flaſk, *fiaſco*. Fuillet, *feuillette*. Gallon, *ſetier*. Grain, *primi*. Gros, *drachme*. Hemine, *chopine*. Hogſhead, *muid*. Karat, *carat*. Letcht, Leth, *laſt*. Load, *cargo*, *ſalma*, *voye*.

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[r] *Thus, a* ſhock . . *of wood is* 66 *pieces*; . . *of linen*, 60 *auns*— *But, when linen is very narrow, and not creaſt or folded double*; 2 *auns of ſuch linens is reckond but* 1 *aun.*

[s] 7.50 *of the Swediſh* ſchippound *make, in London*, 20 cw.

[t] Wool *is ſold by the arroba of* 30 lb.

[u] *Their weight* for gold and ſilver, in duſt, or in bars, in the Indies, *is as follows: a tomine is worth* 2 *rials; and the tomine weighs* 16 *grains: a caſtillan is* 16 *tomine* 4 *grains*: 6 *caſtillan and* 2 *tomines make* 1 *ounce marc-weight: and* 8 *ounces to* 1 *marc.*

[w] *The Turkey*-cantar *makes about* 120 lb *in London.*

[x] 20 medical *of gold is* 3 *ounces, troy-weight, in London.*

[y] *If the* baleman *be weighd with the lodera; it will weigh* 13 *loderas, and* 112 *drams.*

Milr-oe, owl, *millerolle*. Mingeeble, *mingle*. Mui, *muid*. Oak, *oke*. Peché, *pot*. Penny, *numile*. Pinte, *pot*. Poſſon, *poiſſon*. Punchion, *poinçon*. Pound, *arrata*. Quarte, *pot*. Robe, Rove, *arroba*. Roquille, *poiſſon*. Schreve, *viertel*. Septier, *ſetier*. Setier, *chopine*. Somme, *ſomar*. Staro, *ſtaio*. Stekamen, *ſteckan*. Tierce, *aem*. Tiſchauſera, *ſachie*. Toiſe, *fathom*. Tun, *tonneau*. Verge, *viertel*. Vertel, *viertel*. Virge, *viertel*. Virtule, *viertel*. Voedar, *feoder*. Zambre, *açumbre*.

Proportions [*] *of Meaſures.*

(APPLICATORY.)

Feet.

an Engliſh foot divided into 1000 *parts, Others will be as follows:* at Amſterdam, 942. Antwerp, 946. Bononia, 1204. Bremen, 964. Cairo [a], 1824. Cologn, 954. Conſtantinople [b], 2314. Copenhagen, 965. Dantzic, 944. Dort, 1184. Florence [c], 1913. Francfort, 948. Greek, 1007. Leyden, 1033. Lorrain, 958. Mantua, 1569. Mechlin, 919. Middleburg, 991. Naples [c], 2100. Paris [d], 1068. Prague, 1026. Rhinland [e], 1033. Riga, 1831. Roman: *pes colotianus*, 967: *on the monument of Coſſutius*, 970: *on that of Statilius*, 972: *on a congius of Veſpaſian*, 986. Spain, 1001. Straſburg, 920. Toledo, 899. Turin, 1062. Venice, 1162.

YARDS.

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[*] The Proportions *being fixt as underneath, it will be eaſy (thereby) to find an anſwer to other queſtions relating to the following articles : e. g. — What is the value of* 1 vara *of Spain?* (Anſw.) 3.70 *qrs of a yard. for, As* 108 *varas, to* 100 *yards; So* 1 *vara, to* 3.70 *qrs yd. —What number of Engliſh* yards *anſwers to* 100 *varas of Spain?* (Anſw.) 92.59 *yards. for, As* 108 *varas, to* 100 *yards; So* 100 *varas, to* 92.59 *yards.—What is the value of one* ell *of England?* (Anſw.) 1.25 *yards. for, As* 80 *ells, to* 100 *yards; So* 1 *ell, to* 1.25 *yards.*

[a] *calld the* Derſch.

[b] *calld the* Pike, *or* Pico.

[c] *calld the* Brace, *or* Braccio.

[d] *a* Toiſe *is* 6 *feet.*

[e] *or* Leyden *foot; by which moſt of the Northern nations go.*

Yards

100 *Yards of England* make

Arſcheens——140.35, *of* Ruſſia.

Braces——80, *of* Ferrara——104.33, *of* Bergamo, Bologna——130.90, of Ancona——136, *of* Mantua, Modena, Venice: *for ſilk.*——136.26, *of* Milan: *for cloth.*——154.75, *of* Florence, Leghorn, Lucca——171.33, *of* Milan: *for ſilk.*

Canes——38.69, *of* Leghorn——40, *of* Naples, Sicily——40.66, *of* Genoa——44, *of* Rome——46.66, *of* Marſeilles, Montpelier——50, *of* Guienne, Tholouſe——55.66, *of* Barcelona——57.33, *of* Andaluſia——61.19, *of* Avignon, Dauphiné, Provence——97.33, *of* Valencia.

Cavados——133.33, *of* Portugal.

Ells——77.22 *of* Lions——78, *of* France——80, *of* Ferrara, Geneva, London, Oſnabrug——114.60, *of* St Gall: *for woollen*——125, *of* Cambray, Douay——126, *of* Bruges——130, *of* Ypres——131, *of* Artois——131.66, *of* Brabant, Narva——132, *of* Ruremond——133.33, *of* Holland, Norimberg——134, *of* Antwerp——135, *of* Dunkirk, Middleburg——139, *of* Guelderland, Maeſtricht——144, *of* Biſcany (for ſilk) Tournay——146, *of* Denmark——148.66 *of* Norway——149.33, *of* St Gall *for linen*——150, *of* Dantzick——151, *of* Liege——153.85, *of* Pruſſia——156, *of* Sweden——156.25, *of* Naumberg——160, *of* Baſil, Bern, Cologn, Francfort, Hamburg, Koningſberg, Leipſic, Lubec, Riga.——166.60, *of* Breſlaw.

Picos——*of* Candia *(for Silk)* 162.68 *(for Cloth)* 152.68——*of* Turkey *(for Camlets and Grograms)* 150 *(for Linen)* 66.66 *(for Woolen)* 133.33.

Ras——16.30, *of* Piedmont.

Shocks——1.40, *of* Poland.

Varas——81.33, *of* Portugal——108, *of* Spain——109½, *of* Granada.

Veſhcoves——8.77, *of* Ruſſia.

(CUBIC)

Corn

10.25 *Quarters of England* make

Acklings——27, *of* Delf and Rotterdam——88, *of* Schoonhoven.

Aſnées——14.72, *of* Lions.

Buſhels——38, *of* Bourdeaux.

Fanegas——157, *of* Spain.

Hoedts——7.50, *of* Bruges.

Laſt——1, *of* Amſterdam.

Muids——10.50, *of* Bruxelles——4.58, *of* Ghent——27, *of* Amſterdam——33, *of* Groeningen.

Raiſers——18, *of* Dunkirk.

Sacks——28, *of* Dort——40, *of* Middleburg.

Scheppels——30.75, *of* Dantzic——56, *of* Pruſſia——83, *of* Hamburg.

Setiers——17.53, *of* Calais——19, *of* Paris.

Viertels——37.50, *of* Antwerp.

(ITINERARY)

A Mile [f] *contains Rhinland feet:*

in Arabia, 6187. Britain, 5454. Burgundy, 6000. Egypt, 25000. Flanders, 6666. France, 5250. Germany, *ſmall*, 20000: *middle*, 22500: *largeſt*, 25000. Holland, 8000. Lithuania, 18500. Muſcovy, 3750. Perſia, 18750. Poland, 19850. Scotland, 6000. Spain, 7090. Sweden, 30000.

Proportions of Weights.

100 lb *averdupois of England* make:

at Amſterdam, 91 lb 8 oz. Ancona, 136. Andaluſia, 102: *of wool weight* 92. Antwerp, 96 8. Biſcany,

90

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[f] *For, ſuch diſtinction of diſtances, as has been made by ſeveral nations, for the uſe of traveling; by whatever names they are calld:* leagues, paraſangs, verſts, *&c.*

90 (of iron) 78 (of other things). Bologna, 125. **Bremen,** 94 4. Candia *(groſs)* 84 12. *(ſuttle)* 131 9. **Caſtile,** 103 8. Catalonia, 102. Dantzic, 106 12. **Denmark,** 92. Ferrara, 133 5. Florence, 138 14. **Francfort,** 89 7. Geneva, 81 7. Genoa, 137 4. **Granada,** 104 2. **Hamburgh,** 93 5. Leghorn, 132 11. Leipſic, 96 1. Liege, 96 5. Lions, 106. **Mantua,** 138 12. **Milan,** 153 11. Modena, 138 12. Naples, 154 10. **Norway,** 92. Paris, 90 8. Poland, 114 5. Portugal, 104 13. Provence, 113. Pruſſia, 116. Rochelle, 90 9. **Rouen,** 88 Ruſſia, 90. Savoy, 121 13. Spain, 97 4. Tholouſe, 107 11. Valencia, 92 12. Venice, 152. Vienna, 81 7.

for Brabant, V. *Antwerp.* Hall, Leipſic, Holland, *Amſterdam:* Marſeilles, *Provence.* Naumberg, *Leipſic.* Norimberg, *Francfort.* Seville, *Spain.*

TABULATING.

to SUBTRACT, MULTIPLY, DIVIDE, *by Addition.*

1. Twíce-double-Multiplicánd facits †, évery múltiplicator. † *gives the facit of* [a]
2. Tábulate Divíſor: Quote néxt-Digit-únder: Subtráct by Addition [b].

[a] In the MULTIPLICATION-ſum (I) The facits of the multiplicand *twice doubled*, are, as they ſtand againſt the digits 2 and 4. Then, To multiply the multiplicand——into 8 (the laſt figure of the multiplicator) double the facit of the digit 4—into 6 (the 2d figure &c.) add the facit of 4 to that of 2 (=6)—into 7 (the next figure &c.) add together the facits of 1, 2, 4 (=7) placing each of them, as in the common method of multiplication.

		Multipli-cand *cator*	
Digits.	1	98765×768	
	2	197530	(I)
	4	395060	
		—— *by*	
Facits.		790120	8
		592590	6
		691355	7
		75351520 × 768	

In

Divi-dend	*-ſor.*		
75851520 ÷	768	1	
{673794}	1536	2	
5898	2304	3	
43·	3072	4	
	3840	5	*tabulated*
Remainders	4608	6	
	5376	7	
	6144	8	
Quotient: 98765	6912	9	

b In the Division-ſum, here indented, (1) *Tabulate the diviſor*, as in the example, viz. againſt the digit 2, by adding the diviſor to itſelf; againſt 3, by adding together the totals of 2 and 1; againſt 4, by adding the total of 2 to itſelf, or that of 3 to that of 1; and, in like manner, in the reſt, by adding together the totals of any 2 or more digits, equal to the digit whoſe total is ſought. Then, (2) *Quote* (or, for the quotient, take) the *digit* againſt the total *next* leſs, or *under* the firſt correſponding figures of the dividend, viz. 7585.. Then, inſtead of ſubtracting, according to the common method, the facit of the diviſor by 9 (viz. 6912) from (7585) the correſponding figures of the dividend (3) *Subſtract by addition*, and ſay [not, 2 from 5, and there remains 3; but] 2, and (ſo much as will make 5, viz.) 3 is 5: then 1, and (as much as will make 8, viz.) 7 is 8: then 9, and [what will make 15 (ſince 9 cannot be taken from 5) viz.] 6 is 15*, then 1, that I borrow, and 6 is 7: and ſo on.

98765 ÷	968	1
19.29	1936	2
102 *Quotient*.		

In the Division-ſum, in the margin, it appears that——All the tabulating neceſſary to find the quotient, is only to double the diviſor: for, the total next leſs than (the 1ſt dividend) 987, is 968; therefore quote 1: then (the 2d dividend) 196 has no total leſs; therefore quote 0: then the next total leſs than (the 3d dividend) 1965, is (the 2d total viz.) 1936; therefore quote 2.——And, in like manner, may be tabulated any ſum, by ſteps, as there ſhall be occaſion.

* See Diviſion, note [d] †.

in FELLOWSHIP, &c.

FE'LL. *Tarif*:] Stóck by Loſs-Gain: and ínto the Quótient, each Claimant [a].

a *For Example*——I. Two or three hundred perſons having *gaind* (or *loſt*) 8 per cent; it is requird to know

How this muſt be ſhar'd among them, in proportion to the intereſts each-one has in the ſtock? (Anſw.) Having found (according to Fellowſhip, note [b]) What 1 l will gain, at the rate of 8 l per cent, [100 : 8 : : 1 : .0800] viz. .0800 l; and; conſequently, What 1 s will gain [.0800 ÷ 20 = .040] viz. .040 s: a tarif may thence be drawn up to any number; or found occaſionally, as in the following example: Suppoſe any-one had put-in £ 19978 10 3; he muſt have

.0800	×	10000 l: viz. - - - -	0800.0000
		9000 - - - - - - - -	720.0000
		900 - - - - - - - -	72.0000
		70 - - - - - - - -	5.6000
		8 - - - - - - - -	.6400
.040	× -	10 s - - - - - - - -	.400
.003	× -	3 d - - - - - - - -	.90
			1598.2890

that is £ 1598 5 10 1.——II. A man turns *bankrupt* for 80000 l; and his effects are valued at no more than 9000 l. The queſtion is, How-much each creditor is to have in proportion to his debt? (Anſw.) Having found, as in the preceding example, that, for 1 l, a creditor is to have but .1125: If his debt was 566 l;

Then	.1125 × 500 l	=	56.2500
he muſt	60	-	6.7500
have but	6	-	.6750

——III. Suppoſe a diſtrict, which payd, laſt year, 8426 l; is to pay, this year, 12864 l, for *taxes:* and it is requird to find How-much each inhabitant is to pay, in proportion to what he payd the year before? (Anſw.) Say, 12864 : 8426 : : 1 : .6500. Then, He, that payd 36 l (laſt year) muſt

pay	.6500 × 30 l, viz. -	19.5000
(this	—— × 6 - - ,	3.9000
year)		
£ 23 8	- - - - - - -	23.4000

V. Fiſher's conciſe arithmetic p. 244—258.

TARE,

TARE, TRET, CLOUGH.

1 TARE, TRET, CLÓUGH, when from *Gróſs*-weight [a] dedúcted; remains the *Net*-weight.

2 *Táre*'s pounds for package [b]. *Trét*, o in azo [c]. *Clóugh*, e in azy [d].

3 I'nto allówance by quántity; ŏr, by' aliquóts of allowance,
4 Groſs gives *tare* [e]: Tare-net gives *tret* [f]: and Trét-net the *clough* gives [g].

a *Groſs*-weight is that of goods with their duſt, droſs, &c; as alſo with the bag, box, &c, they are in.

b *Tare* is the allowance given for the weight of the bag, box, cheſt, or other package of a commodity. This allowance is ſo much in the hundred-weight, more or leſs, according to the nature of the package.

Commonly there is an allowance (*in* 112 *lb*) of —4, in feathers, hops, wool——6, in iron and latten-wire—8, in brimſtone, copper—10, in copperas—12, in alum, ſalt petre, tallow—14, in almonds, argol, figs, &c.—16, of currants, prunes &c; in caroteels, butts, &c.—18, of oil, in uncertain caſks, &c.

By the book of Rates, in ſeveral commodities, the allowance for tare is not reckond per cent: but ſo-much of the groſs, calld *invoice tare*. Thus—in *Madder*: When the tare is to be deducted; the rule is: 28 lb per Bale.—in *Oil* (1) from Candia: 29 lb per Barrel (2) from New-England: 50 lb per Barrel *—in Silk: per Bale (1) 16 lb, for 3 cw, and upward (2) 14 lb, from 2 cw weight to 3 (3) 12, from 2 cw downwards. ——in *Raiſins*: 14 lb per Frail.——in *Sugar*, from India, in caſks and caniſters, $\frac{1}{6}$: in cheſts and caſks, $\frac{1}{5}$—in Virginia-*Tobacco*, all hogſheads (1) under 3 cw, allow 70 lb tare (2) from 3 to 4 cw, 80 lb (3) from 4 to 5 cw, 90 lb (4) from 5 cw, upward, 100 lb.

* *In Uncertain caſks* it is weighd; and the tare allowd is 18 lb per cw: which, being deducted, is computed at 7 $\frac{1}{2}$ lb per gallon.

c *Tret* is an allowance of 4 lb in 104 (viz. $\frac{1}{26}$) for réfuſe of duſt, dirt, &c. in ſome ſorts of goods,

d *Clough*, or *Draught*, is an allowance of 2 lb in 100 lb (viz. $\frac{1}{50}$) * for the turn of the ſcale; that the commodity may hold-out weight, when ſold by retail †.

* At every 300 weight, in the port of London, 2 lb is uſually allowd for argol, cinamon, cloves, galls, mace, madder, ſumach, tobacco, cotton-yarn, cotton-wool, and other things that have waſte.

† *All theſe Allowances*, beyond ſea, are calld the COURTESIES OF LONDON; becauſe they are not practisd in any other place.——*Beſide theſe allowances* there are alſo others, not ſo common: as (1) BREAK, at ſo-much per bag, barrel, &c. and (2) DAMAGE, at ſo-much in the whole.

e *For example*: Cw 45 3 15 = 5139 *l* × 16 (the allowance) = 82224 ÷ 112 (the quantity, out of which the allowance is made) = 734 lb; that is cw 6 2 6 $\frac{16}{112}$ or $\frac{1}{7}$: Or, cw 45 3 15 ÷ 7 (the $\frac{1}{16}$th'of 112) = cw 6 2 6 $\frac{1}{7}$.

f *For example*: Cw 45 3 15 — 6 2 6 (tare) = 39 1 9 (the tare-net; calld alſo ſuttle) ——Then 39 1 9 (tare-net) ÷ $\frac{1}{26}$ (or, its factors 2 × 13) of 104 = 1 2 1 (tret) which ſubtracted from 39 1 9 (the tare-net) gives 37 3 8 (the tret-net.)

g *Thus* Cw 37 3 8 (the tret-net) ÷ $\frac{1}{50}$ (or, its factors, 5 × 10) of 100 = 3 lb: which ſubtracted from 37 3 8 (the tret-net) gives 37 0 8 (the clough-net.)

Operations of this nature will ſtand, moſt commodiouſly, thus:

	Cw:	45	3	15	Groſs
[a] 16	7	6	2	6	Tare
		39	1	9	net [d]
[b] 26	2	19	2	18	
	13	1	2	1	Tret
		37	3	8	net [e]
[c] 50	5	7	2	7	
}	10	0	3	0	Clough
Aliquets.	Divs	37	0	8	net [f]

[a] 16 is the $\frac{1}{7}$ of 112.

[b] 26 (or, its factors, 2 × 13) is $\frac{1}{4}$ of 104.

[c] 50 (or, its factors, 5 × 10) is $\frac{1}{2}$ of 100.

[d] Tare-net is the remainder of groſs, after tare is deducted.

[e] Tret-net is the remainder of tare-net, after tret is deducted.

[f] Clough-net is the remainder of tret-net, after clough is deducted.

APPENDIX.

A PRACTISE TABLE

Commodity, &c.	Farthings			Pence									Shillings									Commodity, &c.
	PRICE:			1			2			4			1			2			4			
	s	*d*	*q*	*L*	*s*	*d*	*L*	*s*	*d*	*L*	*s*	*d*	*L*	*s*	*d*	*L*	*s*	*d*	*L*	*s*	*d*	
2			2			2			4			8		2			4			8		2
3			3			3			6		1			3			6			12		3
4		1				4			8		1	4		4			8			16		4
5		1	1			5			10		1	8		5			10		1			5
6		1	2			6		1			2			6			12		1	4		6
7		1	3			7		1	2		2	4		7			14		1	8		7
8		2				8		1	4		2	8		8			16		1	12		8
9		2	1			9		1	6		3			9			18		1	16		9
10		2	2			10		1	8		3	4		10		1			2			10
20		5			1	8		3	4		6	8	1			2			4			20
30		7	2		2	6		5			10		1	10		3			6			30
40		10			3	4		6	8		13	4	2			4			8			40
50	1		2		4	2		8	4		16	8	2	10		5			10			50
60	1	3			5			10		1			3			6			12			60
70	1	5	2		5	10		11	8	1	3	4	3	10		7			14			70
80	1	8			6	8		13	4	1	6	8	4			8			16			80
90	1	10	2		7	6		15		1	10		4	10		9			18			90
100	2	1			8	4		16	8	1	13	4	5			10			20			100
COL-	1			2			3			4			5			6			7			-UMN

Here WITHOUT DIVISION you have
by . . . . *Inſpection* . . . . only

PENCE-TABLES, *Groat-tables, &c:* Thus, from the margin (1) in column 2, you read 20d is 1 s 8 d, &c. (2) in column 4, 20 groats is 6 s 8d &c. V. p. 8 *

PRACTISE: Thus——*If one* lb of beef coſt 4 d; What will 40 lb come-to? Carry your eye from 40, in the margin, to the 4th column that has 4 d at the top; and you will find the anſwer, s 13 4 *. ——*What'll one* lb of beef coſt? or, At what rate per pound is the beef ſold, for 30 lb of which the butcher charges me 10 s? Go-up from the price, found (as afore) in the 4th column, to the price of 1 at the top: That is the anſwer; viz. 4 d.——*In the ſame manner* are found anſwers in

PROPORTION, or the *Rule-of-three:* Thus, If 20 coſt 6 s 8 d, 70 will come to L 1 3 4.

by . . . . *Accommodation* . . . . thus:

Lay a blank paper (or ſlate) acroſs the table, under the quantity in the margin; and ſet-down the ſeveral anſwers as given by *Inſpection only.* The Total you will have (I) either *by Addition only:* e. g. (1) The price of 50 oz, at s 1 4 1 an ounce, is (by inſpection) l 2 10 and s 16 8, and s 1 0 2: in all l 3 7 8 2. (2) The price of 50 oz, at s 3 6 2 an ounce, is (by the rates, a top, 2 and 1, viz. 3 s) 5 l, and l 2 10; (by the rates 4 and 2, viz 6 d) s 16 8, and s 8 4; (by the rate of 1 q, doubled) s 2 1: in all l 8 17 1. (3) The price of 100 lb, at the rate of s 4 2 3, for 7 lb (by the rates of 4 d 2 d 1 d, and 1 q, againſt 7 in the margin) will be (at the rates in the ſame columns, againſt 100 in the margin) l 3 0 5. (II) or *by Multiplication and Addition:* e. g. The price of 220 cw, at l 2 7 7 3 per cw, is (by multiplying the prices of 100 by 2) 440 l; and (by adding the prices of 20) l 84 2 1: in all l 524 2 1.

* *NB.* The term, that correſponds to the queſtion-term, is to be of the ſame name. V. Rule-of-three, line 3.——Therefore, If the queſtion-above were propounded thus: What is the price of 5 ſtone, at 4 d a pound; the higher name [ſtone] muſt be reduc'd, by multiplication, to the lower, viz pounds; and the queſtion will be as above.—So likewiſe, If 1 lb coſt l 12 13 7; 1 cw 6 lb (or 118 lb) will be found to come to l 1496 2 10.

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